

# Modelling and Teaching of Magnetic Circuits

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**Abstract** – In the analysis of magnetic circuits, reluctances are commonly modeled as resistances and magnetomotive forces modeled as electromotive forces. In this way the magnetic flux and flux density can be determined easily. However, a close examination should reveal that it is fundamentally wrong to use a resistance, which is a lossy component, to model reluctance, which is a lossless component. This confusion may result in serious problems in the analysis and computer simulation of mixed electric and magnetic circuits. The gyrator approach of modeling can resolve the problem. However, due to the unwillingness of electrical engineers to accept the term gyrator, it has never been widely used. In this paper we propose to revitalize the gyrator model and to further develop it into a z-parameter model. Since z-parameters are well-defined parameters of the 2-port networks, engineers/students in the electrical engineering discipline should find the z-parameter model much easier to understand and to manipulate. In addition, we also propose some strategies to introduce the z-parameter model of magnetic circuits into the curriculum of electrical engineering courses.

**Keywords** - Power converter, resonant converter, motor drives, vector control

## I. INTRODUCTION

In the analysis of magnetic circuits, often reluctances are modeled as resistances and the magnetomotive forces modeled as electromotive forces. While this approach of modeling has been well accepted and widely used by practicing engineers, the concept of modeling reluctance, which is a reactive component, by resistance, which is a lossy component, is fundamentally wrong. This problem becomes very serious when computer simulations are involved. Consider the example of a theoretically lossless inductor. If the reluctance of the magnetic core is modeled as a resistance in a computer program, unexpected (false) loss will appear in simulations involving energy and power.

In 1948, Tellegen proposed the concept of gyrator [1] as a circuit element. This concept was applied to the modeling of magnetic components to solve the problem mentioned above by Buntenbach [2], [3], [4], Hamill [5], [6] and Eaton [7], [8]. However, due to the unwillingness of electrical engineers to accept the term gyrator; this approach of modeling has never been widely used.

The objective of this paper is to revitalize the gyrator concept of modeling magnetic circuits and to further develop it into a form that is readily acceptable to electrical engineers and students, and appropriately simulated by computers. More exactly, we propose to use

a z-parameter two-port network to model the behavior of the interface between an electric circuit and a magnetic circuit [11]. It is assumed that the topic of z parameters is well covered in electrical engineering courses.

It is found that the z-parameter model is particularly useful for the simulation and analysis of integrated magnetics, which are now popularly used in modern power electronics, e.g, polyphase voltage regulator modules for high-speed CPUs. In Section II of this paper, the logic and methodology of using a z-parameter two-port network to model the interface between an electric circuit and a magnetic circuit will be described. In Section III, some examples of the application of the z-parameter model will be given. In Section IV, the duality relationship between the z-parameter model and the equivalent-circuit model of magnetic components will be examined. In Section V, some suggestions on the teaching of magnetic circuits to electrical engineering students will be given.

## II. USE OF Z-PARAMETER TWO-PORT NETWORK TO MODEL THE INTERFACE BETWEEN AN ELECTRIC CIRCUIT AND A MAGNETIC CIRCUIT

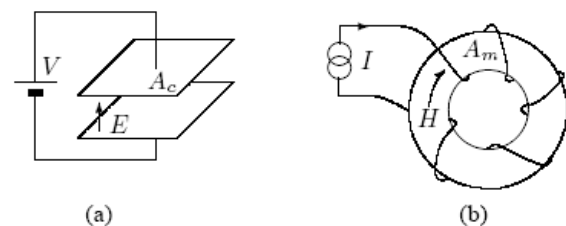


Fig. 1. (a) Simple capacitor. (b) Simple inductor.

To start with, let us consider the electrical parameters of a simple capacitor and the magnetic parameters of a simple inductor, as shown in Fig. 1.

For the capacitor shown in Fig. 1(a), we have

$$EA_c = \frac{Q}{\epsilon_0 \epsilon_r} \quad (1)$$

where E is the electric field strength between the parallel metal plates of the capacitor,  $A_c$  is the area of the metal plates, Q is the charge on the capacitor,  $\epsilon_0$  is the absolute permittivity of space, and  $\epsilon_r$  is the relative permittivity of the dielectric material between the metal plates of the capacitor. For the inductor shown in Fig. 1(b), we have

$$HA_m = \frac{\phi}{\mu_0 \mu_r} \quad (2)$$

where H is the magnetic field strength in the magnetic core,  $A_m$  is the cross sectional area of the magnetic core,  $\phi$  is the total magnetic flux,  $\mu_0$  is the absolute permeability of space, and  $\mu_r$  is the relative permeability of the core material.

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Comparing (1) and (2), it appears natural that for simulation purposes the magnetic field strength  $H$  could be modeled as the electric field strength  $E$  and the magnetic flux  $\Phi$  could be modeled as the electric charge  $Q$ . Based on this assumption, we can find the duality relationship between electric and magnetic parameters as shown in Table 1.

**Table 1: The duality relationship between electrical and magnetic parameters**

Magnetic Parameters	Electrical Parameters
Magnetic field strength $H$	Electric field strength $E$
Magnetomotive force $mmf$	Charge $Q$
Ampere turns	
The rate of change of magnetic flux $\frac{d\phi}{dt}$	The rate of change of charge $\frac{dQ}{dt}$ or current $i$

Now let us consider the electric/magnetic interface shown in Fig. 2(a). Assume that a two-port network with  $z$ -parameters, as shown in Fig. 2(b), is used to model the interface. Port 1 is to model the electric part of the interface and Port 2 is to model the magnetic part. The current  $i_2$  in Port 2 is to model the  $\frac{d\phi}{dt}$  of the magnetic core.

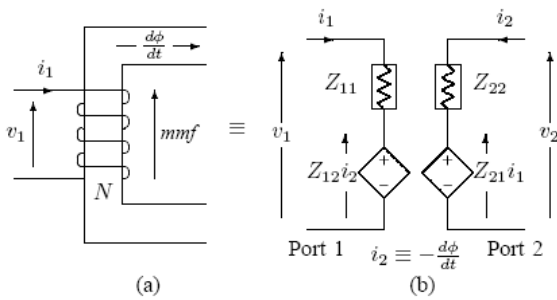


Fig. 2. (a) Interface between electric circuit and magnetic circuit. (b)  $Z$ -parameter two-port model of (a).

$$i_2 \equiv \frac{d\phi}{dt} \quad (3)$$

The negative sign in (3) is due to the opposite directions of  $\frac{d\phi}{dt}$  and  $i_2$  as defined in Fig. 2. The dependent voltage source  $Z_{12}i_2$  is used to model the back  $emf$ .

$$Z_{12}i_2 = \frac{d\phi}{dt} \quad (4)$$

Comparing (4) with (3), it is found that  $Z_{12} = -N[\text{Ohms}]$  (5)

Note that numerically  $Z_{12}$  is equal to  $N$ . But the unit of  $Z_{12}$  is Ohms.

The parameter  $Z_{11}$  can now be determined by assuming  $\frac{d\phi}{dt} = 0$ , or  $i_2 = 0$ . Under this condition, we have from Fig. 2(b)

$$Z_{12}i_2 = 0$$

$$Z_{11} = \frac{V_1}{i_1} = R \quad (6)$$

where  $R$  is actually the series resistance of the winding. The dependent voltage source  $Z_{21}i_1$  is used to model the  $mmf$ . Hence

$$Z_{21}i_1 = Ni_1 \quad (7)$$

Therefore we can find  $Z_{21}$  from (7)

$$Z_{21} = N [\text{Ohms}] \quad (8)$$

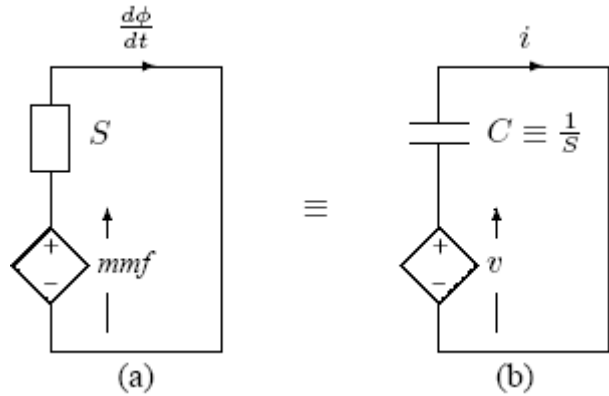


Fig. 3. Modelling the behavior of Port 2 (a) magnetic circuit. (b) electric circuit.

In order to determine  $Z_{22}$ , we assume that a current  $i_1$  is forced into Port 1, and Port 2 is terminated by a magnetic short circuit (magnetic material with zero reluctance) as shown in Fig. 3(a). The  $\frac{d\phi}{dt}$  in the magnetic circuit can

$$\text{then be found as} \quad \frac{d\phi}{dt} = \frac{d(mmf)}{dt} \frac{1}{S} \quad (9)$$

where  $S$  is the reluctance of the magnetic core. If the magnetic core has a magnetic path length of  $l$ , the reluctance of the magnetic core  $S = \frac{l}{\mu_0\mu_r A_m}$ . The behavior of Port 2 can be modelled by the electric circuit shown in Fig. 3(b) where

$$i = \frac{dv}{dt} C \quad (10)$$

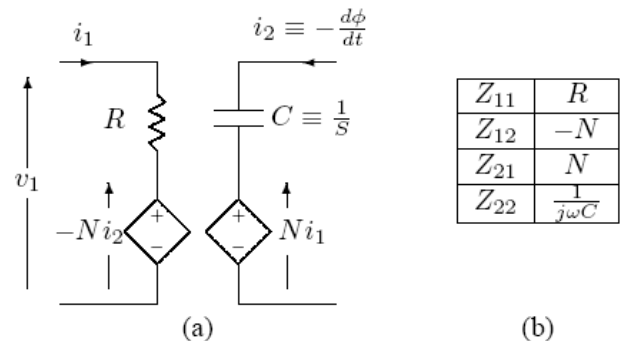


Fig. 4. (a)  $Z$ -parameter model of electric/magnetic interface. (b) Numerical values of  $z$ -parameters.

A term-by-term comparison between (9) and (10) shows the following duality relationship:

$$i \equiv \frac{dv}{dt} \quad (11)$$

$$v \equiv mmf \quad (12)$$

$$C \equiv \frac{1}{S} \quad (13)$$

Based on (5), (6), (8) and the Port 2 model shown in Fig. 3(b), we can derive a complete z-parameter two-port network model of the interface as shown in Fig. 4.

Using the interface model shown in Fig. 4, the z-parameter models of magnetic components can be derived easily. Fig. 5 shows an example of an inductor, where  $R$  is the resistance of the windings,  $S$  is the reluctance of the magnetic core, and  $C$  is a capacitance modelling the behavior of the magnetic core. Fig. 6 shows an example of a transformer, where  $R_p$  and  $R_s$  are the primary and secondary winding resistances respectively. It should be noted that the z-parameter model closely resembles the physical structure of the magnetic circuit. This makes the modelling of complex integrated magnetics much easier.

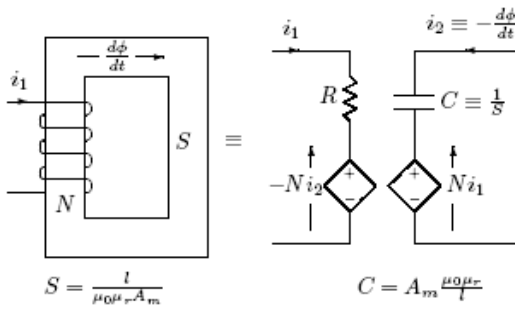


Fig. 5. Modelling of an inductor.

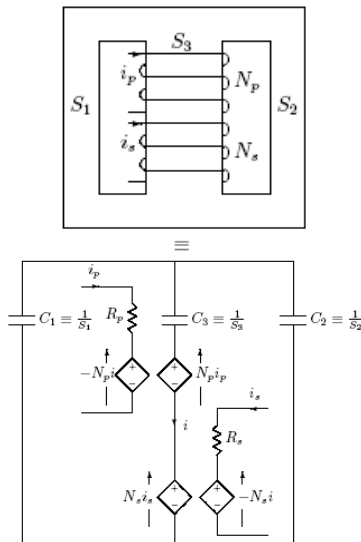
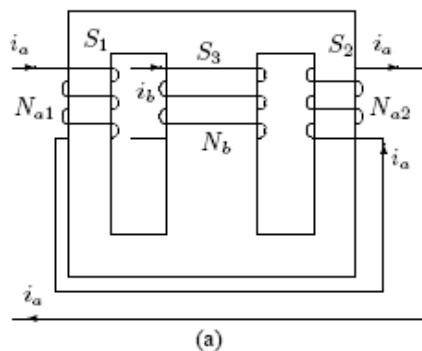
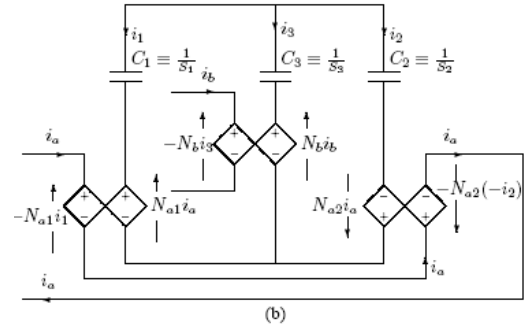


Fig. 6. Modelling of a transformer.



(a)



(b)

Fig. 7. (a) Winding arrangement of an integrated inductor/inductor. (b) Zparameter model of (a).

### III. EXAMPLES OF APPLICATION OF Z-PARAMETER MODEL IN INTEGRATED MAGNETICS

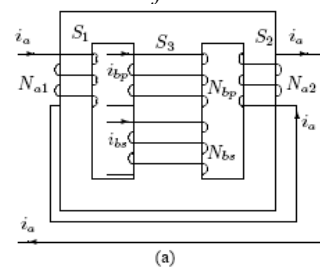
In modern power electronics, integrated magnetics could provide a means to achieve demanding requirements in various applications, e.g., polyphase power converters for high-speed CPUs. The proposed z-parameter model is particularly useful for the analysis of integrated magnetics. Three examples of integrated magnetics and their z-parameter models will be studied in this section.

#### A. Integrated Inductor/Inductor

Fig. 7(a) shows a possible winding arrangement of an integrated magnetics consisting of two inductors. Inductor a, formed by windings  $N_{a1}$  and  $N_{a2}$ , is wound on the outer limbs of the magnetic core. Inductor b, formed by winding  $N_b$ , is wound on the center limb of the magnetic core. By modelling each electric/magnetic interface as a z-parameter two-port network, a z-parameter model of the integrated magnetics can be found, as shown in Fig. 7(b). Note that for the sake of simplicity, the winding resistances of the inductor windings are assumed to be negligible so that the  $Z_{11}$  in the z-parameter model can be neglected. Starting from here, the same assumption will be made in the rest of this paper.

A simple analysis on the circuit shown in Fig. 7(b) will show that if  $C_1 N_{a1} = C_2 N_{a2}$ , the capacitances  $C_1$ ,  $C_2$ , and voltage sources,  $N_{a1}i_a$ ,  $N_{a2}i_a$  will form a balanced bridge. We shall then have  $i_1 = -i_2$ . Under this condition  $i_3$  will be unaffected by  $i_a$ . What this means physically is that the current in Inductor a will produce zero magnetic flux in the center limb of the magnetic core. Thus the operation of Inductor b will not be affected by the current in Inductor a. Under the same condition of  $C_1 N_{a1} = C_2 N_{a2}$ , the current in Inductor b will produce equal but opposite voltages on the two windings of Inductor a. This results in a zero net voltage across Inductor a, meaning that the current in Inductor b will also not interfere the operation of Inductor a.

#### B. Integrated Inductor/Transformer



(a)

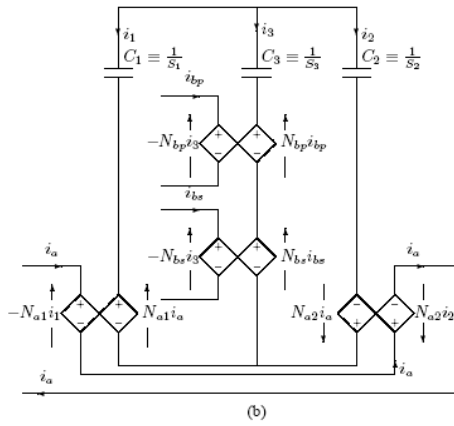


Fig. 8. (a) Winding arrangement of an integrated inductor/transformer. (b) Z-parameter model of (a).

By adding a secondary winding  $N_{bs}$  to the center limb of the integrated inductor/transformer given in Fig. 7(a), an integrated inductor/transformer is formed, as shown in Fig. 8(a). Note that the inductor winding  $N_b$  in Fig. 7(a) now becomes the primary winding  $N_{bp}$  in Fig. 8(a). Here  $N_{bp}$  and  $N_{bs}$  are the two windings of the transformer, and  $N_{a1}$  and  $N_{a2}$  together form the winding of the inductor.

It can be proved that, if  $C_1 N_{a1} = C_2 N_{a2}$ , the operations of the inductor and the transformer will be independent of each other.

### C. Integrated Transformer/Transformer

By adding secondary windings  $N_{a_s1}$  and  $N_{a_s2}$  to the outer limbs of the integrated inductor/transformer shown in Fig. 8(a), an integrated transformer/transformer is formed, as shown in Fig. 9(a). The corresponding z-parameter model of the integrated transformer/transformer is shown in Fig. 9(b). It can be proved that, if  $C_1 N_{a_p1} = C_2 N_{a_p2}$  and  $C_1 N_{a_s1} = C_2 N_{a_s2}$ , the operations of the two transformers will be independent of each other.

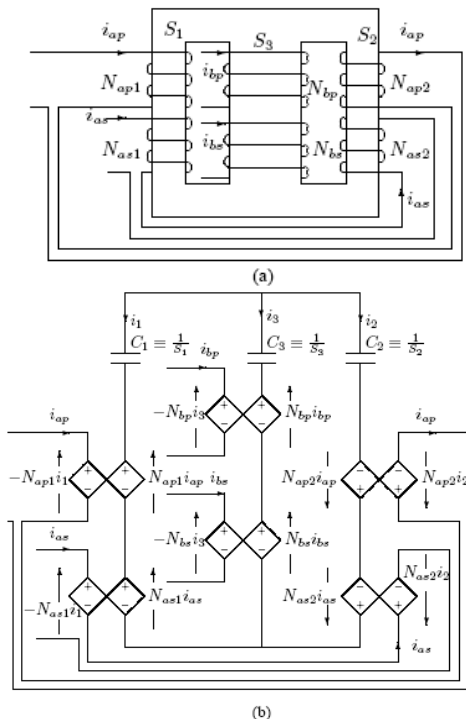


Fig. 9. (a) The winding arrangement of an integrated transformer/transformer. (b) Z-parameter model of (a).

## IV. CONVERSION OF Z-PARAMETER MODEL TO EQUIVALENT-CIRCUIT MODEL

The z-parameter models of magnetic components developed in Section III are very useful for computer simulation and design purposes. However, electronic engineers often find difficulties in understanding the physical meanings of these z-parameter models. It will therefore be helpful if these z-parameter models can also be converted into their equivalent-circuit models.

Based on the example shown in Fig. 10(a), we propose the following steps to derive the equivalent-circuit model of an integrated magnetics:

- 1) By replacing each electric/magnetic interface of the integrated magnetics with an appropriate z-parameter two-port network, develop a z-parameter model of the integrated magnetics first. When this step of operation is performed on the integrated magnetics given in Fig. 10(a), a z-parameter model can be found as shown in Fig. 10(b). Note that, in Fig. 10(b),  $C_{L1}$  and  $C_{L2}$  are added to model the leakage magnetic paths of the magnetic core.
- 2) Take away the parts of the z-parameter model that represent the windings of the integrated magnetics. This results in a model that contains only the magnetic components. The resultant model is referred to as the magnetic model. When this step of operation is performed on the z-parameter model shown in Fig. 10(b), we get the magnetic model as shown in Fig. 10(c).
- 3) Assign a name to each loop in the magnetic model. In this example, the names assigned are A, B, C, D, and E, as shown in Fig. 10(c).
- 4) Assuming that each loop is converted into a node, carry out a duality circuit transformation on the magnetic model shown in Fig. 10(c). After the transformation, we get the duality circuit shown in Fig. 10(d). Note that loops A, B, C, D, and E in Fig. 10(c) now become nodes A, B, C, D, and E in Fig. 10(d).
- 5) Replace each of the current sources in the duality circuit shown in Fig. 10(d) by an ideal  $N : 1$  transformer, where  $N$  is the number of turns of the winding concerned. When this step of operation is performed on the magnetic model given in Fig. 10(d), we get the resultant equivalent-circuit model as shown in Fig. 10(e). It is interesting to note that the leakage magnetic paths  $C_{L1}$  and  $C_{L2}$  in Fig. 10(b) have now become leakage inductances  $L_{L1}$  and  $L_{L2}$  in Fig. 10(e).

It should be understood that, as far as the external behavior is concerned, the z-parameter model shown in Fig. 10(b) and the equivalent-circuit model shown in Fig. 10(e) are identical. Both models can be used for circuit analysis and computer simulations.

Sometimes it may be convenient for one of the windings to be referred to as the primary winding. If this is the case, the ideal transformer of the primary winding may be replaced by a pair of wires connecting the output to the

input (effectively an ideal 1:1 transformer without galvanic isolation). All inductances should then be scaled by the factor  $N_p^2$ , where  $N_p$  is the number of turns of the primary winding. Also the transformer ratios should be scaled to  $\frac{N_s}{N_o}$  where  $N_s$  is the

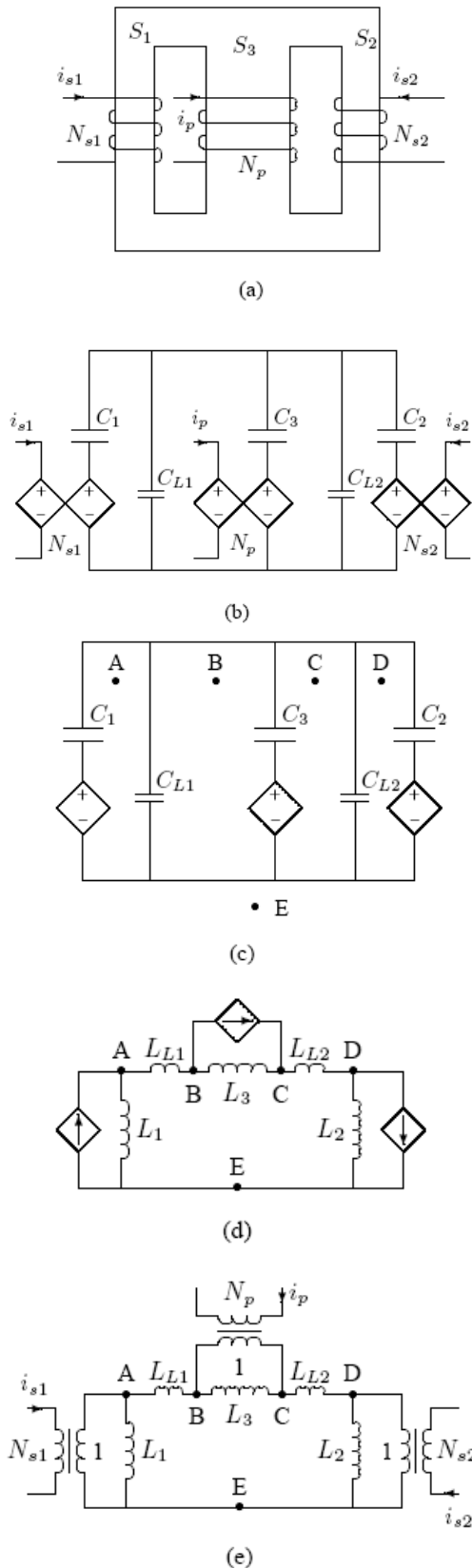


Fig. 10. (a) Winding arrangement of an integrated magnetics. (b) Z-parameter model. (c) Magnetic model. (d) Duality circuit. (e) Equivalent-circuit model of (b).

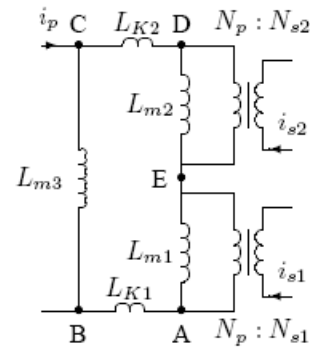


Fig. 11. Alternate form of equivalent-circuit model.

number of turns of the secondary winding concerned. When this conversion is performed on the equivalent-circuit model given in Fig. 10(e), assuming  $N_p$  to be the primary winding, we get an alternate form of the equivalent-circuit model of the integrated magnetics, as shown in Fig. 11. Note that in Fig. 11,  $L_{m1} = N_p^2 L_1$ ,  $L_{m2} = N_p^2 L_2$ ,  $L_{m3} = N_p^2 L_3$ ,  $L_{K1} = N_p^2 L_{L1}$  and  $L_{K2} = N_p^2 L_{L2}$ .

### V. TEACHING OF MAGNETIC CIRCUITS TECHNICAL INFORMATION

The z-parameter/grator model of magnetic circuits has been extensively used in our research works on integrated magnetics [9], [10], [11], [12], [13], [14]. It has been proved to be an extremely useful tool. However, a lot of efforts are required in order to promote the use of the z-parameter/gyrator model, because engineers/students have not been exposed to it when they study magnetic circuits. It appears that, in order to fundamentally solve the problem, we have to reconsider the way the topic of magnetic circuits is taught in universities.

In the teaching of magnetic circuits, traditionally reluctance is modelled as resistance and *mmf* is modelled as *emf*. In this way, the flux and flux density of a magnetic circuit can be easily calculated. This approach of teaching is helpful to enable students to “visualize” how magnetic flux “flows” in a magnetic circuit. It is also our believe that such an intuitive understanding is necessary when students encounter magnetic circuits the first time. However, students should be warned that there are actually deficiencies in modelling reluctance (which is a lossless component) as resistance (which is a lossy component). Keeping this in mind, students would be better prepared to accept the z-parameter model to be introduced later.

As far as timing is concerned, the z-parameter model should be introduced in the curriculum whenever there is a need for mixed simulation of electric and magnetic circuits. In this way students should readily appreciate the real reason for introducing the z-parameter model. Example simulations involving magnetic components used in modern power electronics circuits, taking into account various leakage magnetic paths, could be used to convince students of the power of z-parameter models. The conversion of z-parameter model to equivalent-circuit model, as described in Section IV, should be useful to help students develop the equivalent circuits of complex magnetic components and to visualize the effects of leakage magnetic paths.

## VI. CONCLUSION

Based on the gyrator model, the z-parameter model of magnetic circuits has been developed. The advantages of the z-parameter model include

- 1) true modelling of the behavior of mixed electric and magnetic circuits;
- 2) easy acceptance by engineers and students in the electrical engineering discipline; and
- 3) close resemblance to the physical structure of magnetic components, making the modelling of complex integrated magnetics and leakage inductances much easier.

Some suggestions on how the z-parameter model should be introduced into the curriculum of electrical engineering courses are given. Hopefully the introduction of the z-parameter model will solve a long-standing problem in the modelling, analysis, and simulation of magnetic circuits.

This section describes the contribution of the paper so that even if readers have not read the body of your paper, they still understand the main idea of the paper. You should not insert any discussion statements in this section because they can be fitted in the previous sections. Even if the author only designed and tested a system, he can also state the achievement in this section. The following statement is an example: The theory has been implemented in an electronic circuit. The circuit has been prototyped and tested. The experimental results agreed very well with the theoretical prediction and verified the theory proposed.

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