A Mathematical Model for IGBT

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Abstract - A mathematical model has been developed for an IGBT by compartmenting it into two diodes, which are connected in series with reverse configuration. One diode is an ordinary diode while other is a controlled diode. The performance of the controlled diode depends upon the magnitude of gate emitter voltage and collector emitter voltage. A gate emitter voltage greater than gate emitter threshold voltage modulates the conductivity of controlled diode together with hole injection from the p+ base near the collector terminal. Parameters of these diodes are estimated by applying curve fitting tool and power invariance method. The validity of the model has been checked by a simulation example. The results of simulation show appreciable closeness with actual one.

Keywords - Insulated Gate Bipolar Transistor (IGBT), Gate Emitter Threshold Voltage, Collector Emitter Threshold Voltage, Conductivity Modulation, Gate Emitter, Collector Emitter

List of Symbols

a_o	=	*Coefficients(dimensionless)*
a_1	=	*Coefficients* (V^{-1})
b	=	*Incremental wattage of IGBT which raises temperature by 1°C*
C_{ix}	=	*Nonlinear capacitance between gate and emitter (Farad)*
C_C	=	*Shunt capacitance of diode* D_1 *(Farad)*
C_e	=	*Shunt capacitance of diode* D_2 *(Farad)*
i_C	=	*Collector current(ampere)*
I_{d1}	=	*Current source(ampere)*
$I_o(T)$	=	*Reverse saturation current of diode* D_1 *at Temperature* T^o *K,(ampere)*
I_o	=	*Reverse saturation current of diode* D_1 *at Temperature* 0^o *K,(ampere)*
k	=	*Boltz mann constant, (Joule /°K)*
m	=	*Modulation factor (dimensionless)*
M	=	*Mass of IGBT (gram)*
Q	=	*Electronic Charge (coulomb)*
R_c	=	*Resistance of diode* D_1 *(ohm)*
r_e	=	*Variable resistance of diode* D_2 *(ohm)*
S	=	*Equivalent specific heat (joule/gm °C)*
T	=	*Temperature of the junction of diode D ,(°K)*
V_g	=	*Gate emitter voltage (V)*
V_{d1}	=	*Constant voltage source (V)*
V_{d2}	=	*Variable voltage source (V)*
V_d	=	*Voltage between collector and emitter (V)*

V_{d1} = *Constant voltage source (V)*
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V_{GET}	=	*Gate emitter threshold voltage (V)*
V_{CET}	=	*Collector emitter threshold voltage (V)*
V^+	=	*Potential at junction of two diodes (V)*
V_T	=	*kT/q, (V).*

I. Introduction

Ever since the evolution of IGBT a number of models have been reported in literature namely IG SPICE, PSPICE [1-3], analytical [4] and EMTP [5]. Most of them explain both steady state and dynamic behavior of IGBT with varying degree of accuracy. In utility related applications such as flexible ac transmission systems (FACTS) [6], requiring high power electronics, the system is generally described by means of differential/algebraic equations which are solved using any one of the several higher level computer languages such as FORTRAN, C, etc. For such cases the detailed model of the devices will result a large computation time, loss of accuracy and an overwhelming amount of information, which is difficult to digest [7-8]. In order to keep the execution time within reasonable limit, complex devices in power electronics system such as IGBT etc. may be represented by approximate models, which may be represented by standard algebraic equations [9-12]. An improved and simplified EMI modeling method considering the IGBT switching behavior model has been proposed in [13] where device turn-on and turn-off dynamics are investigated by dividing the nonlinear transition by several stages.

This paper proposes a simple mathematical model of IGBT, consisting of only diodes that are described by well-known algebraic equations. The model is illustrated with controlled diode characteristics whose performance depends on a modulation factor *m*, which is function of the voltages applied at different points across the device. The parameters of this model are obtained by applying power invariance [14] and curve fitting technique [15]. The model is tested for an existing IGBT in the lab extracting the required data from the instruction sheet.

II. Proposed Model

Fig. 1 shows the basic architecture of an IGBT. It is compartmented into two parts by segmenting with dotted lines as shown near the junction J_1 in *n-* base region. By inspection, the part of IGBT, between the dotted line and collector terminal forms a *p-n* junction diode D_1.

The remaining part of IGBT, which accommodates the gate and emitter, can be viewed as a controlled diode, D_2.

The paper first received 22 Sept. 2006 and in revised form 31 Mar 2007.
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Between the junction of two diodes and gate terminal, a parallel combination of RC circuit of constant values is added. This RC circuit is attributed because of the insulation resistance and capacitance of SiO_2 layer underlying the gate terminal. A non-linear capacitance C_{ix}, between gate and emitter is also added which gives intermediate circuit of IGBT as shown in Fig. 2.

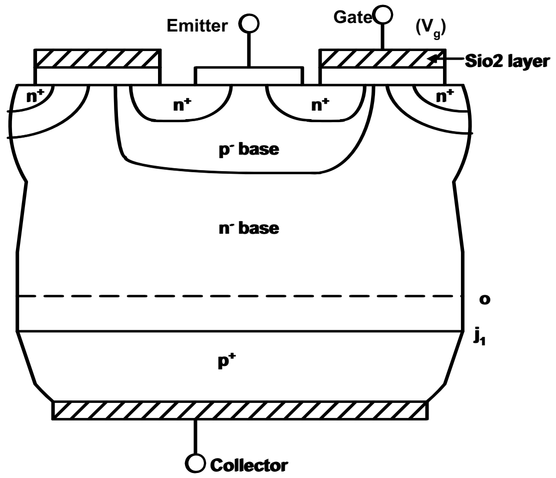

Fig. 1: Basic Construction of IGBT

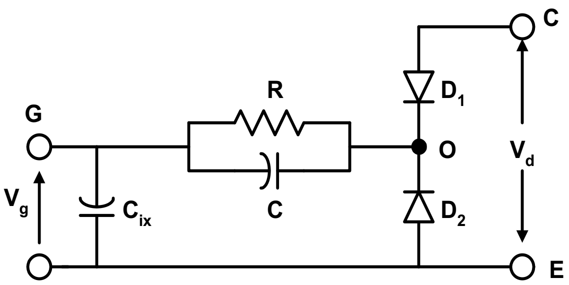

Fig. 2: Inter-mediate circuit of IGBT

The final model can be obtained by replacing the diodes by their equivalent circuit as shown in Fig. 3.

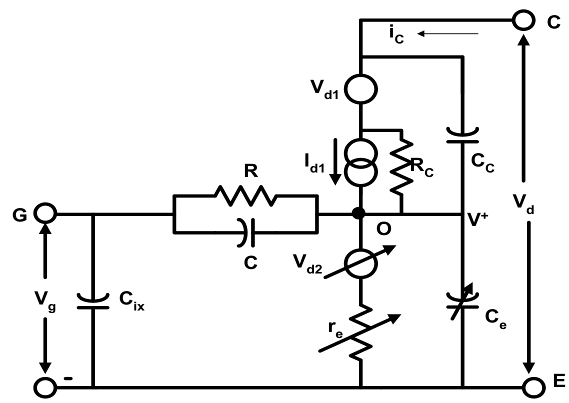

Fig. 3: Steady- State Model of IGBT

In this model, the ordinary diode has been replaced by a constant voltage source V_{d1} in series with a parallel combination of a current source, I_{d1}, a resistance R_c and a shunt capacitance C_C. The controlled diode is modeled as a variable capacitance C_e in parallel to a series combination of variable voltage source V_{d2} and resistance r_e.

At low frequencies all these capacitances become open circuit and the resultant circuit becomes the steady state DC model of IGBT as shown in Fig. 4.

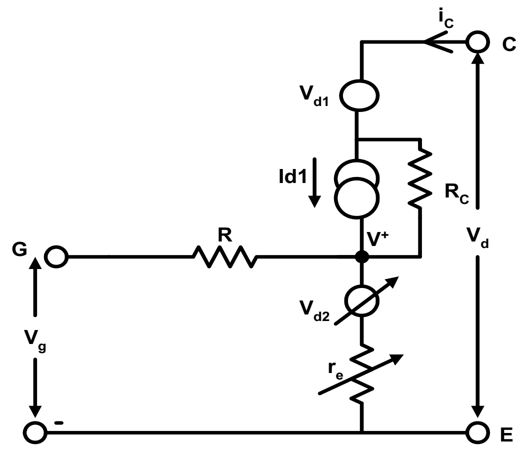

Fig .4: DC Model of IGBT

III. Parameter Identification

In order to identify the parameters of the proposed model, two strategies are resorted: 1) the principle of invariance of power in the components and 2) mathematical tool of curve fitting. Let a positive voltage V_d is applied at collector with respect to emitter. Also a voltage $V_g > V_{GET}$ is applied at gate terminal.

If V_+ is the potential of point 'o' which is junction of two diodes then the voltage $V_d - V_+$ will appear across the diode D_1. If i_C is the current flowing through the circuit, then the voltage-current relation of diode D_1 can be expressed as,

$$i_C = I_o(T)\{e^{\frac{V_d - V_+ - V_{d1}}{V_T}} - 1\} \quad (1)$$

Rearrangement of equation (1), gives,

$$V_d - V_+ = V_{d1} + (kT/q) \ln [1 + i_C/I_o(T)] \quad (2)$$

Differentiation of equation (2) with respect to i_C gives the resistance of the diode

$$R_C = \frac{d(V_d - V_+)}{d(i_C)} = \frac{kT}{q}(i_C + I_o(T)) \quad (3)$$

To identify the current source, principle of power invariance is applied to the diode D_1 and its equivalent. Power consumed by diode D_1 is given by,

$$(V_d - V_+)\, i_C = (V_{d1} + (kT/q) \ln [1 + \frac{i_C}{I_o(T)}]\, i_C \quad (4)$$

and its equivalence is given by

$$(V_{d1} + r_C . i_C - r_C . I_{d1})\, i_C \quad (5)$$

Power invariance requires

$$(V_{d1}+\frac{kT}{q})\ ln\ [1+\frac{i_C}{I_o(T)}]\ i_C$$

$$= (\ V_{d1} + R_C.i_C - R_C\ I_{d1})\ i_C \tag{6}$$

Above equality should hold for all value of i_C This implies that differentiation of (6) for both side with respect to i_C does not affect the equality or

$$\frac{d}{d(i_C)}\left(\frac{kT}{q}\ln(1+\frac{i_C}{I_o(T)})i_C\right) =$$

$$\frac{d}{d(i_C)}\big((R_C i_C - R_C I_{d1}))i_C\big) \tag{7}$$

Substituting the value of R_C and dR_C/di_C in equation (7) and simplifying it gives

$$\frac{d}{d(i_C)} + \frac{I_{d1}}{i_C} - \frac{I_{d1}}{i_C + I_0(T)} =$$

$$1 - \frac{I_{d1}}{i_C + I_0(T)} - (1+\frac{I_0(T)}{i_C})\ ln\ (1+\frac{i_C}{I_0(T)}) \tag{8}$$

or $$I_{d1} = i_C - (i_C + I_o(T))\ ln\ (1+\frac{i_C}{I_0(T)}) \tag{9}$$

In order to investigate the mathematical description of controlled diode D_2 a functional relationship between i_C, V_g and V_d is required. Since IGBT model has been approximated to consist of mainly diodes, therefore, it is appropriate to describe collector emitter voltage drop equal to product of ordinary diode voltage drop multiplied by a modulating factor *m* which should of course be function of collector emitter voltage and gate emitter voltage. Thus, overall, IGBT can be described with a modified diode characteristics as given below

$$V_d = m(V_g, V_d)\ V_T\ ln\ (1+\frac{i_C}{I_0(T)}) + V_{CET} \tag{10}$$

Thermo-dynamic equation governing the temperature [16] of IGBT is given by

$$M.S.\ \frac{dT}{dt} + b(\ T - T_0) = V_d\ i_C \tag{11}$$

For steady state analysis $\frac{dT}{dt} = 0$ therefore from (11)

$$T = T_0 + \frac{V_d i_C}{b} \tag{12}$$

Substituting the value of T in (10) and rearranging the terms we get

$$V_d = \frac{\frac{mkT_o}{q}\ln\left(1+\frac{i_C}{I_o}\right)+V_{CET}}{1-\left(\frac{mki_C}{qb}\right)\log\left(1+\frac{i_C}{I_o}\right)} \tag{13}$$

From (13) it is clear that for very large value of $V_{d,}$ denominator of equation (13) approaches to zero. This implies that

$$m\ (V_g) = \frac{bq}{ki_C \log\left(1+\frac{i_C}{I_o(T)}\right)} \tag{14}$$

Equation (14) reveals that *m* becomes sole function of V_g for large value of V_d. That is what IGBT characteristics shows at higher value of V_d. From equation (13), *m* can be written as follows:

$$m(V_g,\ V_d) = \frac{V_d - V_{CET}}{\left(\frac{kV_d i_C}{qb}+\frac{kT_o}{q}\right)\log\left(1+\frac{i_C}{I_o(T)}\right)} \tag{15}$$

Basically IGBT is dual input single output device. The input ordered pair (V_g, V_d) corresponds to the output i_C. Thus equation (15) transforms above mentioned correspondence into (V_g, V_d) and $m\ (V_g, V_d)$.

IV. Model Validation

To identify *m* as function of (V_g, V_d) a table is generated from the characteristics sheet of IGBT [1] by extracting the data required for the model. For 4 distinct values of V_g and 7 distinct values of V_d the value of *m* is calculated from the equation (15) knowing corresponding value of i_C. This covers almost all the operating range of IGBT. Table 1 is thus obtained which gives the different parameter of IGBT. From this table it is observed that as the value of V_d is increased, *m* increases correspondingly whereas for an increase of V_g, *m* decreases. To obtain exact relation between *m* and V_d for a given value of V_g, the method of least square curve fitting is used. Let *m* be approximated by one-dimensional polynomial given by equation (16)

$$m = a_o + a_1\ V_d \tag{16}$$

Then, for V_g equal to 7, the coefficients a_o and a_1 will be calculated as -0.714 and 1.115 respectively and the above polynomial will be given by equation (17)

$$m = 1.115\ V_d - 0.714 \tag{17}$$

Similarly, for V_g equal to 8, 9 and 10 the above polynomial may be written, respectively, as

$$m = 0.852\ V_d\ - 0.526 \tag{18}$$
$$m = 0.502\ V_d + 1.098 \tag{19}$$
$$m = 0.346\ V_d\ + 1.3748 \tag{20}$$

The coefficients a_o and a_1 are plotted in Fig. 5 for different values of $V_{g.}$ It is observed that if these points are joined together it will result a straight line, which can be extended on both axis. These lines will provide the value of coefficients a_o and a_1 for any other value of V_g not calculated in the above equations.

Now if the value of a_o and a_1 are known for a specified value of V_g, then the value of *m* may be calculated for a specified value of V_d and $V_{g.}$ using (16). Once the value of *m* is known, the current i_C is directly calculated from (13).

Table 1: Various Parameters of IGBT

	V_d =1.0	V_d =2.0	V_d = 4.0	V_d = 6.0	V_d = 8.0	V_d =10.0	V_d =20.0
V_g =7	i_C= 0.5 m=0.392	i_C= 2 m=1.633	i_C= 3.33 m=4.056	i_C= 3.33 m=4.456	i_C= 3.33 m=8.794	i_C= 3.33 m=11.069	i_C= 3.33 m=21.497
V_g =8	i_C= 0.9 m=0.385	i_C= 5.5 m=1.58	i_C= 12.2 m=3.677	i_C=12.2mm = 5.58	i_C= 12.2 m=7.31	i_C= 12.2 m=8.87	i_C=12.2 m=14.84
V_g =9	i_C= 0.95 m=0.384	i_C= 6 m=1.57	i_C=16.4 m=3.45	i_C= 23.3 m=4.78	i_C= 25 m=5.89	i_C= 25 m=6.91	i_C= 25 m=10.27
V_g =10	i_C= 1 m=0.384	i_C= 6.25 m=1.57	i_C=20.8 m=3.34	i_C= 34.2 m=4.19	i_C= 39.6 m=4.83	i_C= 40.5 m=5.46	i_C=40.5 m=7.49

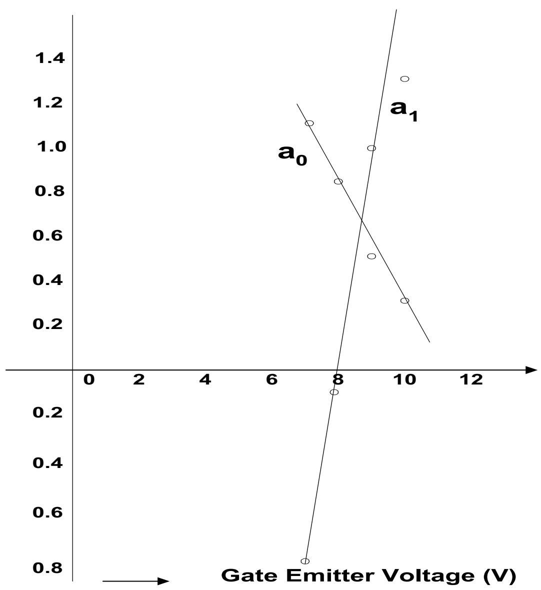

Fig. 5: Relation between V_g & a_o and V_g & a_1

The proposed model is compared with the models available in reference [9]. Table 2 shows the value of *m* calculated from (16) for a specified value of gate and collector voltage and compared with the value obtained in Table 1 for a known value of i_C. As data are extracted from the characteristics sheet of IGBT, therefore it is inevitable that there is an error due to scaling factor and other manual factor. The results are appreciably close in the face of this avoidable error.

V. Conclusions

A simple model for an IGBT has been developed which consists of only two diodes. One diode is an ordinary diode D_1 and other diode is a controlled diode D_2. The characteristics of IGBT have been approximated as modified diode characteristics, where the total voltage drop across collector and emitter is equal to the product of an ordinary diode voltage drop and a modulating factor m.

This modulating factor, *m*, has been established as the function of collector emitter voltage and gate emitter voltage. A one-dimensional polynomial approximation has been used to identify the value of modulating factor, m. In order to identify the modulating factor and in turn the circuit parameters, power invariance and curve-fitting technique has been used. For curve fitting technique, data are extracted from the data sheet of IGBT available in lab. The value of modulating factor obtained from the polynomial and that extracted from the data sheet are in closed agreement. The proposed model is simple accurate and integrates the DC behavior into a single formula.

Table 2: Calculated value of modulating factor m

V_g	V_d	i_C	m	m (calculated)	Error%
7	4	3.33	4.05	4.0	3.16×10^{-4}
7	12	3.33	13.28	12.96	2.4
8	6	12.2	5.5751	5.571	6.4×10^{-4}
9	2	5.5	1.570	1.5702	-4×10^{-4}
10	20	40.0	7.49	7.4905	-4×10^{-4}
10	6	34.2	4.19727	4.1973	8.32×10^{-2}
11	8	47.7	4.3907	4.0446	7.88
12	6	45.0	3.744	3.4141	8.81

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BIOGRAPHY

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