Harmonics Elimination in Multilevel Inverter with Two Unequal Voltage Batteries

Y.R. Manjunatha[1] M.Y. Sanavullah[2]

Abstract – Elimination of harmonics in a multilevel inverter with two unequal dc sources is considered. That is, for a given fundamental output voltage, our aim is to find the switching angles, that produce the fundamental while not generating specifically chosen harmonics. A procedure is given to find all sets of switching angles for which the fundamental is produced while lower order harmonics are eliminated. This is done by first converting the transcendental equations that specify the elimination of the harmonics into an equivalent set of polynomial equations. Then, using the mathematical theory of resultants, solution to this equivalent problem is found.

Keywords – Multilevel inverter, harmonics elimination, unequal voltage batteries

I. Introduction

Multilevel inverters provide more than two voltage levels. A desired output voltage waveform can be synthesized from the multiple voltage levels with less distortion, at low switching frequency, higher efficiency, and lower voltage rating devices. An important issue in designing an effective multilevel inverter is to ensure that, the total harmonic distortion (THD) in the output voltage waveform is small [4]. This requires an (mathematical) algorithm to determine when the switching should be done so as not to produce harmonics and a fast real-time computing system to implement the strategy. A method was reported in [2] and [3] that presented a procedure the switching angles for the H-bridges in a cascaded converter using the mathematical theory of resultants. In that work, a complete solution was presented for computing all possible switching angles that achieved the requisite fundamental voltage and eliminated lower order harmonics. However, it was assumed that the dc sources were all equal, which will probably not be the case in applications even if the sources are nominally unequal. Here, it is shown how the method in [3] can be extended to two un-equal dc source inverter. Specifically, eliminating harmonics in a multilevel converter in which the separate dc sources do not have equal voltage levels is considered. Generally each phase of a cascaded multilevel converter requires n DC sources for 2n + 1 levels. For many applications, to get many separate DC sources is difficult, and too many DC sources will require many long cables and could lead to voltage unbalance among the DC sources. To reduce the number of DC sources required when the cascaded H-bridge multilevel converter is applied to a motor drive, a scheme is proposed in [1] that allows the use of two unequal DC sources to generate 7 level equal step multilevel inverter output instead of three equal batteries.

This scheme provides the capability to produce higher voltages (where they are needed) at low switching frequency, has inherent low switching losses and high conversion efficiency. For electric/hybrid electric vehicle motor drive applications, two H-bridges for each phase is a good tradeoff between performance and cost.

For the required fundamental output voltage, it is desired to find out the switching times (angles) that produce the fundamental and no specifically defined harmonics. In this paper, the lower order harmonics are eliminated making use of two unequal DC voltages for H-bridges.

II. Cascaded H-Bridges

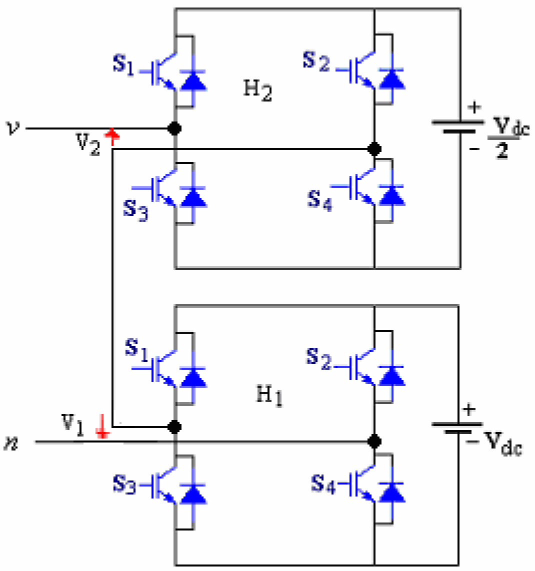

Fig. 1: Single phase multilevel cascaded H-bridge inverter.

The cascaded multilevel inverter consists of a series of H-bridge inverter units. As previously mentioned, the general purpose of this multilevel inverter is to synthesize a desired voltage from several separate dc sources (SDCSs), like batteries, fuel cells, solar cells, and ultra-capacitors. Fig. 1 shows a single-phase structure of a cascade inverter with SDCSs [6]. Each SDCS is connected to a single-phase full-bridge inverter. Each inverter level can generate three different voltage outputs, $+V_{DC}$, 0, $-V_{DC}$ with different combinations of the four switches, S_1, S_2, S_3, and S_4.

The conventional method of generating 7 level multilevel inverter output waveform is, by using three batteries of equal magnitude and three cascaded H-bridges. In this scheme the duty cycle for each of the voltage level is

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different. If this pattern of duty cycle is used on a motor drive continuously, the level 1 battery is cycled on for a much longer duration than the level 3 battery. This means that the level 1 battery will discharge much more than the level 3 battery.

To operate a cascaded multilevel inverter using two unequal DC source, we have proposed to use the first DC sources (i.e., the battery connected to first H-bridge, H_1) as V_{dc} and the magnitude of voltage of second battery as $\frac{V_{dc}}{2}$.Fig. 1 shows cascaded H-bridges multilevel inverter with two unequal batteries. The DC source for the first H-bridge (H_1) is a battery or fuel cell V_1 with an output voltage of V_{dc}, and the DC source for the second H-bridge (H_2) is V_2 with an output voltage of $\frac{V_{dc}}{2}$. The output voltage of the cascaded multilevel inverter is

$$V(t) = v_1(t) + v_2(t) \quad . \tag{1}$$

By applying the triggering pulses to the switches of H_1 appropriately, the output voltage V_1 can be made equal to V_{dC}, 0, or -V_{dC}. while the output voltage of H_2 i.e., V_2 can be made equal to $\frac{V_{dc}}{2}$, 0, or -$\frac{V_{dc}}{2}$. by applying the triggering pulses to the switches of H_2 appropriately.

The 7 output voltages of the inverter are $\left(\frac{V_{dc}}{2}+V_{dc}\right)$, , $\left(\frac{V_{dc}}{2}\right)$, 0, $-\left(\frac{V_{dc}}{2}\right)$, $-V_{dc}$, $-\left(\frac{V_{dc}}{2}+v_{dc}\right)$ which are 7 possible output levels. Fig. 2 shows the 7 level equal step output voltage waveform,

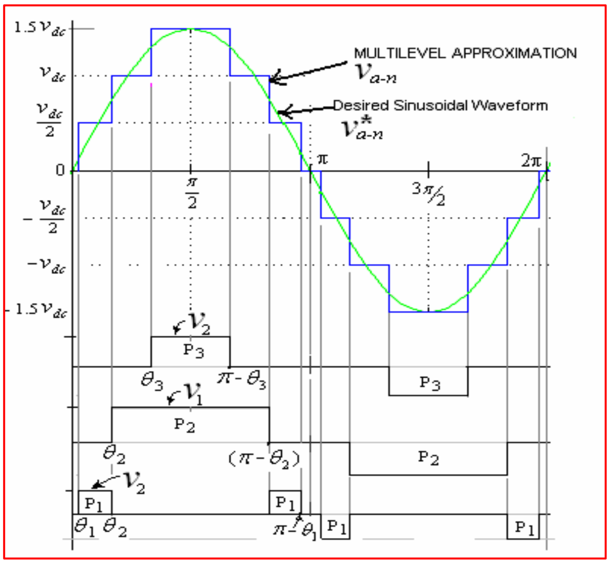

Fig. 2: Seven level equal step output voltage

This problem does not occur for the switching scheme shown above. In this paper a switching scheme is proposed in which v_1=output of $H_1 = V_{dc}$ and v_2=output of $H_2 = V_{dc}/2$ From θ_1 to θ_2, voltage v_2 will appear at the output (i.e. V_{a-n}) and from θ_2 to θ_3, voltage v_1 will appear, from θ_3 to $\theta_{\pi-\theta 3}$ voltages (v_1+v_2) will appear, from $\theta_{\pi-\theta 3}$ to $\theta_{\pi-\theta 2}$ voltage v_1 will appear, and from $\theta_{\pi-\theta 2}$ to $\theta_{\pi-\theta 1}$ voltage v_2 will appear at the out put. Hence both the batteries discharge equally in every half cycle (i.e. for every 90^0).

III. Switching Algorithm for Multilevel Inverter

The Fourier series expansion of the 7-level equal step output voltage waveform is [4]

$$\left.\begin{aligned} V(\omega t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{4V_{dc}}{n\pi}(\cos n\theta_1) \\ +(\cos n\theta_2)+(\cos n\theta_3)\sin(n\omega t) \end{aligned}\right\} \tag{2}$$

where 'n' is the harmonic number of the output voltage. Given a desired fundamental voltage V_1, one wants to determine the switching angles θ_1, θ_2, θ_3 so that $V(\omega t) = V_1 \sin(\omega t)$, and specific higher order harmonics are eliminated [8]-[10]. For three-phase motor drive applications, the triplen harmonics in each phase need not be considered as they automatically disappear in the line-to-line voltages. In this paper, the goal is to eliminate the 5th harmonic and the 7th harmonic can be filtered out. Mathematically, this can be formulated as the solution to the following equations:

$$\left.\begin{aligned} \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = m \\ \cos 5\theta_1 + \cos 5\theta_2 + \cos 5\theta_3 = 0 \\ \cos 7\theta_1 + \cos 7\theta_2 + \cos 7\theta_3 = 0 \end{aligned}\right\} \tag{3}$$

This is a system of three transcendental equations with three unknowns θ_1, θ_2, and θ_3. There are many ways to solve for the angles. One approach to solving this set of nonlinear transcendental equations (3) is to use an iterative method such as the Newton–Raphson method [4]. In this work, the method given in is extended to find *all* solutions to (3). This methodology is based on the mathematical theory of resultants of polynomials which is a systematic procedure for finding the roots of *polynomial* equations. In this method, the set of equations (3) is converted to a polynomial system shown below by setting $x_1 = \cos\theta_1$, $x_2 = \cos\theta_2$, $x_3 = \cos\theta_3$ and using the trigonometric identities

$$\cos 5\theta = 5\cos\theta - 20\cos^3\theta + 16\cos^5\theta$$

$$\cos(7\theta) = -7\cos\theta + 56\cos^3\theta - 112\cos^5\theta + 64\cos^7\theta$$

to transform (3) into the equivalent conditions

$$p_1(x) \triangleq V_1 x_1 + V_2 x_2 + V_3 x_3 - m = 0$$

$$p_5(x) \triangleq \sum_{i=1}^{3} V_i\left(5x_i - 20{x_i}^3 + 16{x_i}^5\right) = 0$$

$$p_7(x) \triangleq \sum_{i=1}^{3} V_i\left(-7x_i - 56{x_i}^3 + 112{x_i}^5 + 64{x_i}^7\right) = 0 \tag{4}$$

Where $x = (x_1, x_2, x_3)$ and $m \triangleq V_f/(4V_{dc}/\pi)$. The modulation index is $m_a = \frac{m}{n} = V_f/(n4V_{dc}/\pi)$. This follows from the fact that each inverter has a dc source that is nominally equal to V_{dc} so that the maximum output voltage of the multilevel inverter is nV_{dc}. Consequently, a square wave of amplitude nV_{dc} results in the maximum fundamental output possible of $V_{f\max} = 4nV_{dc}/\pi$ so that

$$m_a \triangleq V_f / V_{f\max} = V_f/(n4V_{dc}/\pi) = m/n$$

This is now a set of three *polynomial* equations in the three unknowns x_1, x_2, x_3. Further, the solutions must satisfy $0 \le x_3 < x_2 < x_1 \le 1$ [4].

substituting $x_3 = \dfrac{(m - (V_1 x_1 + V_2 x_2))}{V_3}$ into P_5, P_7 leads to,

$$P_5(x_1,x_2) = V_1(5x_1 - 20x_1^3 + 16x_1^5) + V_2(5x_2 - 20x_2^2 + 16x_2^5) + 5V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right) - 20V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right)^3 + 16V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right)^5 \tag{5}$$

and

$$P_7(x_1,x_2) = V_1(-7x_1 + 56x_1^3 - 112x_1^5 + 64x_1^7) + V_2(-7x_2 + 56x_2^3 112x_2^5 + 64x_2^7) - 7V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right) + 56V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right)^3 - 112V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right)^5 + 64V_3\left(\frac{m-(V_1x_1+V_2x_2)}{V_3}\right)^7. \tag{6}$$

IV. Elimination using Resultants

In order to explain how one computes the zero sets of polynomial systems, utilizes *elimination theory* and uses the notion of *resultants*. Briefly, one considers $a(x_1,x_2)$ and $b(x_1,x_2)$ as polynomials in x_2 whose coefficients are polynomials in x_1 [4]. Then, for example, letting $a(x_1,x_2)$ and $b(x_1,x_2)$ have degrees 3 and 2, respectively in x_2, they may be written in the form

$$a(x_1,x_2) = a_3(x_1)x_2^3 + a_2(x_1)x_2^2 + a_1(x_1)x_2 + a_0(x_1) \tag{7}$$

$$b(x_1,x_2) = b_2(x_1)a_2^2 + b_1(x_1)x_2 + b_0(x_1) \tag{8}$$

The $(n \times n)$ *Sylvester* matrix,

where $n = \deg_{x_2}\{a(x_1,x_2)\} + \deg_{x_2}\{b(x_1,x_2)\} = 2 + 3 = 5$ is defined by,

$$S_{a,b}(x_1) = \begin{bmatrix} a_0(x_1) & 0 & b_0(x_1) & 0 & 0 \\ a_1(x_1) & a_0(x_1) & b_1(x_1) & b_0(x_1) & 0 \\ a_2(x_1) & a_1(x_1) & b_2(x_1) & b_1(x_1) & b_0(x_1) \\ a_3(x_1) & a_2(x_1) & 0 & b_2(x_1) & b_1(x_1) \\ 0 & a_3(x_1) & 0 & 0 & b_2(x_1) \end{bmatrix} \tag{9}$$

The *resultant* polynomial is then defined by,

$$r_1(x_1) = \mathrm{Res}\,(a(x_1,x_2), b(x_1,x_2), x_2) \triangleq \det S_{a,b}(x_1) \tag{10}$$

and is the result of solving $a(x_1,x_2) = 0$ and $b(x_1,x_2) = 0$ simultaneously for x_1, i.e., eliminating x_2. This can be referred in [12] – [15] for an explanation of this fact. The computational challenge for this approach is in the symbolic calculation of the determinant of the Sylvester matrix. However, the results in [16], [17] show that this computation can be carried out quite efficiently.

V. Experimental Results

To validate the proposed cascaded H-bridge multilevel motor drive control scheme, a three-phase cascaded H-bridge multilevel inverter has been developed. The switches used for this inverter are the IGBTs GT60M303 (Toshiba Make). The gating pulses are generated by microcontroller board. A 3-phase induction motor is selected with the specifications shown in the table-1 below. Also motor model is developed using MATLAB Simulink.

Table 1: Specifications of the motor

S. No.	Type of the motor	3-ϕ Induction motor
1	Rated output power	3700 watts (5 HP)
2	Rated line-to-line voltage	415 volts
3	Rated current	8.4 amps
4	Number of poles	4
5	Frequency of the supply voltage	50 Hz
6	Rated speed	1485 rpm
7	Type of winding	Y-connected

The parameters of this motor are calculated by conducting No-load test, Blocked Rotor test and Retardation test. This motor model is simulated using MATLAB and the simulated results are compared with that of practical results. It is found that these two results are very close to each other. Then the motor model is simulated with the proposed cascaded multilevel inverter.

Table 2: Results of motor on load test, at rated voltage 415V

S. No.	I_L Amps	W_{in} Watts	T_L N-m	N rpm	I/p Watts	O/p Watts	η %
1	4.5	200	0	1480	200	0	0
2	4.9	900	2.53	1463	900	387.6	43
3	5.5	1600	7.6	1413	1600	1124.5	70.2
4	6.1	2100	10.96	1376	2100	1579	75.2
5	6.8	3100	15.18	1320	3100	2098	67.6

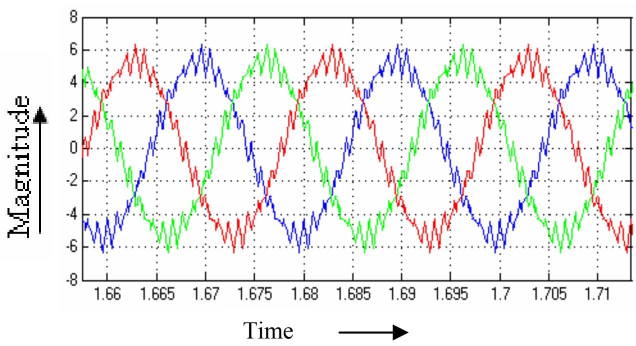

Fig. 3: Stator current, motor fed from SPWM Inverter

Fig. 3 shows the stator currents of three phase induction motor on no-load fed from 2 level inverter. In this fig. even though the waveform looks close to sinusoidal, it has distortion. Fig. 4 is the stator currents of the same motor fed with 7 level inverter. Fig 5 shows the spectrum. It is observed that the distortion is almost reduced. Hence the motor runs smoothly and safely. Fig. 6 is the three phase output voltage of multilevel (7 Level) inverter with two unequal voltages.

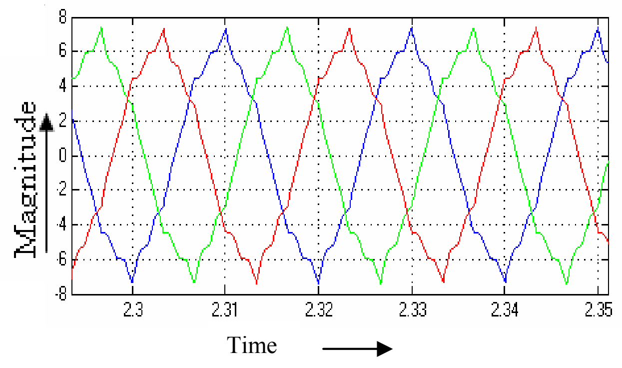

Fig.4: Stator current, motor fed from ML Inverter

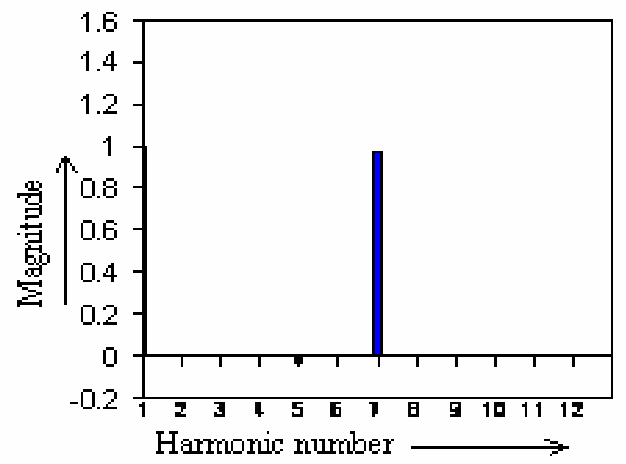

Fig. 5: Harmonic spectrum of Line-to-Line Vg

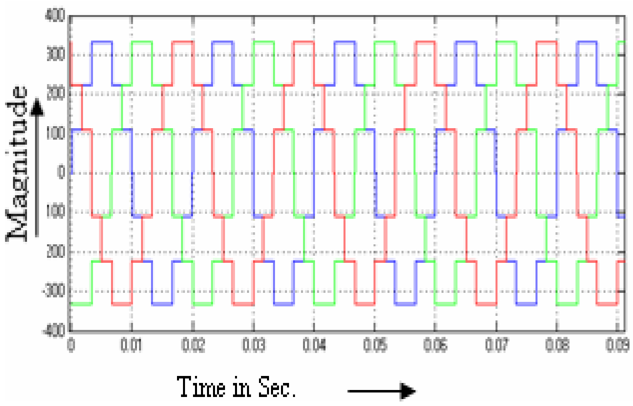

Fig. 6: Output voltage waveform of MLI (3 phase)

IV. CONCLUSION

Elimination theory and the notion of resultants can be used to eliminate the lower order harmonics in a multilevel inverter that has unequal dc sources. This method is expected to have widespread application as most multilevel converters do not have dc sources that are equal. Also as the number of batteries are reduced, the Electrical Vehicle mileage will also be improved. Both the batteries discharge equally in every half cycle (i.e. for every 90^0). By looking into harmonic spectrum, we observe that, all the lower order harmonics are eliminated, only the magnitude of 7^{th} harmonic, whose frequency is very high (350Hz) compared to fundamental frequency (50Hz). Hence it can be easily filtered out using a filter capacitor.

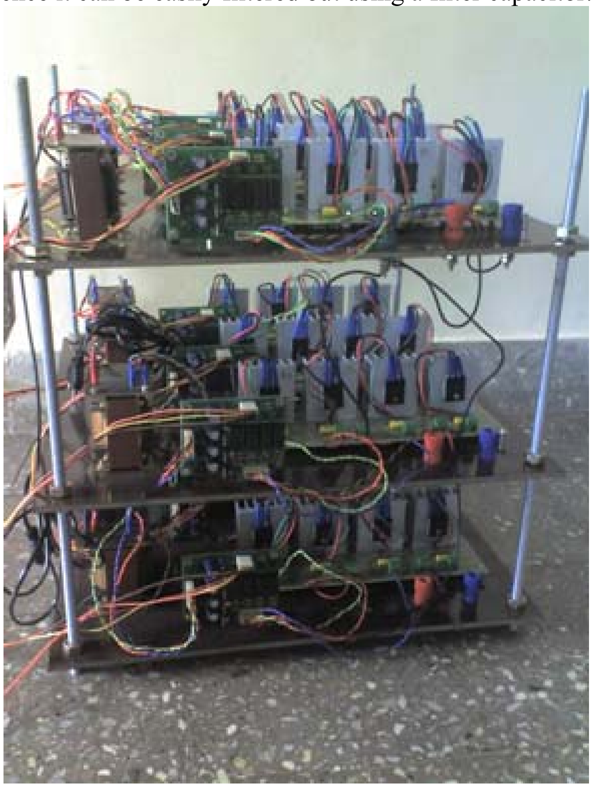

Fig. 7: Photograph of experimental setup

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