# Control Strategy with Game Theoretic Approach in DC Micro-grid

Mingxuan Wu [1] Lei Dong [2] Furong Xiao [3] Xiaozhong Liao [4]

**Abstract–This paper proposes a decentralized control algorithm for DC micro-grid which is based on the game theory without communication, the proposed algorithm is an application of the Nash certainty equivalence principle. Since it is achieved without communication, it benefits from concerned reliability issues. Compared with the droop control system, extensive analysis based on simulations demonstrates that the proposed modeling, although simplified, is sufficient for an adequate control system design. The proposed control algorithm brings more accurate DC bus voltage and good current sharing performance. The simulation results verify the voltage stability and accuracy of DC bus, and current-sharing control of DC-DC converters are achieved simultaneously.**

**Keywords–Nash certainty equivalence principle, DC Micro-grid, game theory, distribute generation**

## I. INTRODUCTION

Nowadays, energy and environmental problems are interrelated with the factors, such as the growth of energy demand, and the high quality with safe and reliable electric requirement. Against the background of these problems, a large number of distributed generations (DGs) are being installed into power systems. However, it is well known that an insertion of many DGs affects power quality of the utility grids, and it may cause problems such as rising of voltage and protection problem. In order to solve these problems, one of the solutions is to construct a new power system, i.e. micro-grids which have been especially researched all over the world [1]. DC distribution micro-grids are suitable for those kinds of energy storages and DGs, which can reduce conversion losses and supply high quality power [2]. There are some advantages using DC micro-grid in practice [3].

In the literature, most control strategies depend on a communication infrastructure. Reference [4] presents the operation of a multi-agent system for the control of a micro-grid. The expensive communication system can be avoided by using active power/frequency droop control in the conventional grid control system [6,7]. An improved droop control principle is presented for the control of generators in an islanded micro-grid [8]. The droop control method is a good solution for load sharing. However, the conventional droop method also has several drawbacks including a slow transient response, a trade-off between load sharing accuracy and voltage deviation [12]. The load sharing accuracy is affected by line, DG output impedances and corresponding voltage drops. To improve the accuracy of load sharing, increased droop gain scheme is adopted. However, increased droop gain has a negative impact on the overall system stability. Moreover, it increases the range of system voltage variations [13-15].

In this paper, a control strategy is proposed based on the game theory which does not depend on communication infrastructure. As is shown in the Fig. 1, the multiple DG system is modeled and employs game theory [9] to design control strategy. The control strategy can minimize output voltage error. It is found that performance of each DG is practically independent in the system.

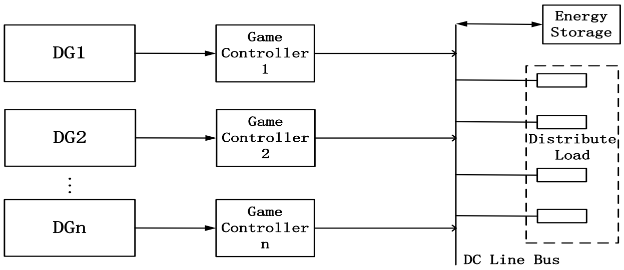


Fig.1: Multi-circuit system without communication

## II. DECENTRALIZED CHARGING CONTROL PROBLEMS FOR PCS IN MICRO-GRID

The decentralized DGs charging control problem is a form of non-cooperative game, where a large number of selfish DGs transfer the electricity resources to the DC line bus to ensure the DC bus voltage stable. This paper presents a novel control strategy that achieves Nash equilibrium by simultaneously updating each DG's best individual strategy with respect to minimize the cost function. The algorithm is unrelated to the number of DGs, since they update their control strategy simultaneously and independently. The proposed decentralized charging control strategy drives the system asymptotically to a unique Nash equilibrium.

The proposed algorithm is an application of the so-called Nash certainty equivalence principle. The control algorithm can achieve optimality of the micro-grid, i.e. it can make the DC bus voltage stable and load sharing. Nash equilibrium is the most important concept in game theory. In fact, Nash equilibrium is a kind of "stalemate" in which no one is interested in changing [16].

Definition 1**:** A collection of control strategies $\left\{d_n^*; n \in N\right\}$ is Nash equilibrium if each individual DG can benefit nothing by changing its own strategy

$$J_n(d_n^*; d_{-n}^*) \le J_n(d_n; d_{-n}^*) \tag{1}$$

for all $d_n \in (0,1)$, and all $n \in N$.

The paper first received 18 Dec 2012 and in revised form 13 Mar. 2013.
Digital Ref: APEJ-2013-04-416
[1,2,3,4] School of Automation, Beijing Institute of Technology, Beijing,
[1] E-mail: wweiy1296@163.com, corresponding author
[2] E-mail: leidong@bit.edu.cn
[3] E-mail: xfrongok@126.com
[4] E-mail: liaoxiaozhong@bit.edu.cn

where $d_{-n}^{*}$ is the collection of control strategies of the whole DGs without DG $n$ [19].

In other words, $d_n^{*}$ is the solution of minimum cost function

$$d_n^{*} \in \arg\min J_n(d_1^{*}, \cdots, d_{n-1}^{*}, d_n^{*}, d_{n+1}^{*}, \cdots, d_N^{*}), \quad n = 1, 2, \cdots, N. \tag{2}$$

Therefore, $d_n^{*}$ is the control strategy which can obtain the minimum value of cost function $J_n(d_n)$.

If the two above-mentioned conditions are achieved by each DG, the micro-grid will achieve the state of Nash equilibrium, i.e. the DC micro-grid will keep voltage stable and load sharing.

*A. Virtual Rivals Strategy*

Assuming there are $n+1$ DGs supply power for the micro-grid which means there are $n$ rivals for every DG. Estimation of supply power strategy for every rival will be difficult to calculate because the workload is extremely large. In this paper, a conception of virtual rivals is introduced which means to take the other DGs as an equivalent virtual rival. This will simplify the estimation for every rival and it has advantages as follows: 1) Simplicity: Reducing the number of rivals from $n$ to 1 simplifies the model and the complexity of calculation greatly, and 2) High estimating accuracy: Estimating supply power strategy for $n$ rivals, since the uncertainty is too much for every rival, so the estimating error is relatively large [5]. When estimating the equivalent rival, the randomness will be offset and the estimating accuracy is enhanced.

By this way, the method simplifies the n-player-game problem to two-player-game problem that simplifies the complexity of problem greatly. Therefore, in the discussion of this paper, we only discuss the two-player-game problem because it has the same result of n-player-game problem.

*B. Proposed Optimal Controller*

The underlying decentralized DGs charging control is a non-cooperative dynamic game. Each DG shares other DGs with the information by DC line bus voltage. So we need formulate the decentralized DGs charging control problem as a dynamic games.

We consider charging control of each DG of size $M$ with the charging time $t = \{0, 1, \cdots, T-1, T, T+1, \cdots, \infty\}$ where $T$ denotes the ultimate charging steadily instant. The serial number of DG is denoted as $M = \{0, 1, \cdots, n, \cdots, N\}$. With a control strategy of micro-grid with game theoretic approach, we assume that DGs achieve Nash equivalence at time $T$. The objective is to implement a collection of DGs charging strategy that achieves dual objectives, 1) the DC line bus voltage achieves setting value of voltage, and 2) each DG achieve the state of current sharing.

The objective of the control is to minimize the output voltage error. Hence, a definition of the cost function of each DG $n$ is given by

$$\begin{aligned} J_n(u,i) &= AP + B\sum_{m=0}^{m=t-1}(k_1u_r - k_2u_m - k_3i_m)^2 + D(u_r - u_t)^2 \\ &= Au_ti_t + B\sum_{m=0}^{m=t-1}(k_1u_r - k_2u_m - k_3i_m)^2 + D(u_r - u_t)^2 \end{aligned} \tag{3}$$

where， $A$, $B$, $D$, $k_1$, $k_2$ and $k_3$ are all non-negative constant, $P$ represents the output power of the each DG, $u$ and $i$ denote the output voltage and current of the each PC, $u_r$ is the voltage reference of the DC line bus.

As is shown in (3), the output current, voltage and error of the system voltage output are included in the cost function. Where $AP$ denotes the output power, which has the minimization of the input to the system results in minimum input energy (minimum cost). Where $B\sum_{m=0}^{m=t-1}(k_1u_r - k_2u_m - k_3i_m)^2$ determines the penalty for deviating from the setting value, and $D(u_r - u_t)^2$ denotes minimization of the output error that is one of the main targets of the system. It will be shown that the presence of the squared deviation term ensures convergence to a unique collection of locally optimal strategies which is Nash equilibrium [11]. The cost incurred in deviating from the voltage reference of the DC line bus.

This paper proposes a control scenario which individual DG minimize its own operating cost function (3). In order to minimize its own operating cost, it is adopted by calculate method of the extreme value of binary function [10].

Therefore, we can demonstrate using the control law (4) and (5) can minimize its cost function (3). The control law can be simplified as:

$$i_r = \frac{2Bk_2}{A}\sum_{m=0}^{m=t-1}(k_1u_r - k_2u_m - k_3i_m) + 2D(u_r - u_t) \tag{4}$$

and

$$k_1 = k_2 + k_3\frac{i}{u_r}. \tag{5}$$

According to the transformation in (4) and (5), through the voltage reference $u_r$ ,the previous voltage $u_m$ and current $i_m$ $(m = 0, 1, \cdots, t-1)$ and present voltage $u_t$ can calculate the reference current $i_r$ which can be obtained, and then detect the output current $i$. Finally, PI adjust is adopted for a current closed loop to ensure a suitable power output. The local information such as output voltage $u$ and output current $i$ can be detected directly. It is obviously that only local information can minimize the cost function. Accordingly, using this method can achieve desired performance in voltage regulation and current sharing among the DGs. Such a structure has notable advantages such as simplicity and reliability. And this method can satisfy the most important characteristics of the micro-grid, i.e. plug and play. DG can be considered as different agents. Assuming each agent is 'rational' which means every agent always following the maximum of its own benefit function. Finally, the micro-grid system achieves the state of Nash equilibrium.

## III. Small-Signal Model of Game Theoretic Strategy

A linear model is necessary for designing linear controls. An average model is sufficient for the bandwidth requirements of typical applications [17]. Accordingly, a simple average model for the buck converter is developed based on ideal transform concepts for switching. The DG's converter shown in Fig. 3 can be replaced by an average model.

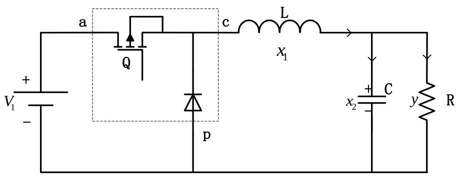


Fig. 3: The module of buck converter

At time $t=[0,d]T$ , the switch $Q$ is turned on,

$$\begin{cases} i_a(t)=i_c(t) \\ u_{cp}(t)=u_{ap}(t) \end{cases} \tag{6}$$

where $d$ is the duty cycle of the switch $Q$ , $i_a$ is the input current of terminal $a$ , $i_c$ is the output current of terminal $c$ . $u_{cp}$ and $u_{ap}$ is the voltage between terminals $a$ , $p$ and $c$ respectively.

At time $t=[d,1]T$ , the switch $Q$ is turned off,

$$\begin{cases} i_a(t)=0 \\ u_{cp}(t)=0. \end{cases} \tag{7}$$

Ideal transformer averaging concepts can be directly applied to buck converter. A large signal average model can be formed with transformers by realizing (6) and (7) and similar average current equations,

$$\begin{cases} \bar{i}_a(t)=\bar{i}_c(t)\times d \\ \bar{u}_{cp}(t)=\bar{u}_{ap}(t)\times d. \end{cases} \tag{8}$$

The small signal model is formed from the large signal model by perturbing the large signal parameters about a particular operating point:

$d=D+\hat{d},\quad \bar{u}_{cp}(t)=V_{cp}+\hat{u}_{cp},\quad \bar{u}_{ap}(t)=V_{ap}+\hat{u}_{ap},$

$\bar{i}_a(t)=I_a+\hat{i}_a,\quad \bar{i}_c(t)=I_c+\hat{i}_c,$

where the set $\{D,V_{ap},V_{cp},I_a,I_c\}$ defines a particular operating point, and the set $\{\hat{d},\hat{u}_{ap},\hat{u}_{cp},\hat{i}_a,\hat{i}_c\}$ defines perturbing. The nonlinear model for the converter operating in the continuous conduction mode is given by

$$\begin{cases} I_a+\hat{i}_a=\left(I_c+\hat{i}_c\right)\times\left(D+\hat{d}\right) \\ V_{cp}+\hat{u}_{cp}=\left(V_{ap}+\hat{u}_{ap}\right)\times\left(D+\hat{d}\right). \end{cases} \tag{9}$$

The dynamic model given in (9) is nonlinear and cannot be directly employed to design a control system with the most commonly used techniques, which require a linear model. However, if the magnitudes of the perturbations are much smaller than their steady-state values, the nonlinear terms can be ignored without resulting in significant error, and a linear approximation of the system is obtained. Assuming dynamic components are much smaller than steady-state components,

$$\frac{\hat{d}}{D}\ll 1,\frac{\hat{u}_{ap}}{V_{ap}}\ll 1,\frac{\hat{u}_{cp}}{V_{cp}}\ll 1,\frac{\hat{i}_a}{I_a}\ll 1,\frac{\hat{i}_c}{I_c}\ll 1. \tag{10}$$

So the infinitesimal of higher order $\hat{i}_c\times\hat{d}$ and $\hat{u}_{ap}\times\hat{d}$ can be neglected. Hence, (10) can be divided into steady-state component and perturbing component respectively.

Steady-state components

$$\begin{cases} I_a=I_c\times D \\ V_{cp}=V_{ap}\times D. \end{cases} \tag{11}$$

Perturbing component

$$\begin{cases} \hat{i}_a=i_c\times D+I_c\times\hat{d} \\ \hat{u}_{cp}=\hat{u}_{ap}\times D+V_{ap}\times\hat{d}. \end{cases} \tag{12}$$

Thus, a small-signal reduced-order linear model for the buck converter can be defined as Fig. 4.

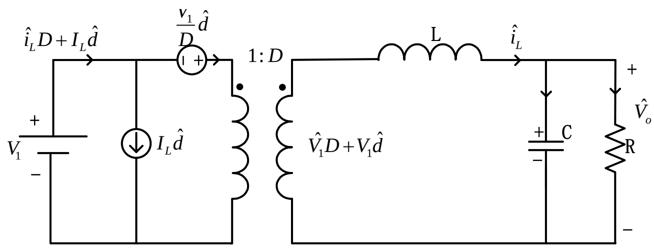


Fig. 4: The small-signal equivalent circuit model of buck converter

**Table 1: Parameters of the small-signal equivalent circuit model.**

| Parameters | Values | Unit |
|---|---|---|
| Output inductor | 10e-2 | *H* |
| Resistance of load | 20 | Ω |
| Output storage capacitor | 300e-6 | *F* |
| Load | 20 | Ω |
| Switching frequency of the converter | 2 | *kHz* |

The parameters of the small-signal model are given in Table 1, the transfer functions (13) and (14) can be easily obtained applying the Laplace transform in (12) assuming zero initial conditions

$$G_{vd}(s)=\frac{\hat{v}_o(s)}{\hat{d}(s)}=\frac{\frac{1}{cs}//R}{\frac{1}{cs}//R+Ls}=\frac{U_{in}}{s^2LC+sL/R+1} \tag{13}$$

$$\begin{aligned} G_{id}(s)&=\frac{\hat{i}_L(s)}{\hat{d}(s)}=\frac{\hat{v}_o(s)}{\hat{d}(s)}\frac{\hat{i}_L(s)}{\hat{v}_o(s)}=G_{vd}(s)\frac{\hat{i}_L(s)}{\hat{u}(s)} \\ &=G_{vd}(s)\frac{1}{\frac{1}{Cs}//R}=\frac{1+sCR_0}{s^2LC+sL/R+1}\times\frac{U_{in}}{R_0}. \end{aligned} \tag{14}$$

Therefore, the open loop small-signal model without game theoretic control is obtained, and the bode plot of $G_{vd}(s)$ is shown as Fig. 5,

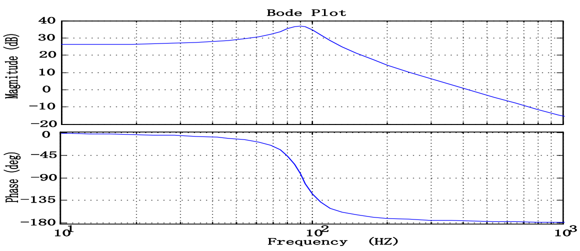


Fig. 5: The bode plot of $G_{vd}(s)$

As is shown in the Fig.5, phase margin is only about 10, the system is in a critical steady state. When the proposed game theoretic control is adopted, the close loop structure is illustrated in Fig.6.

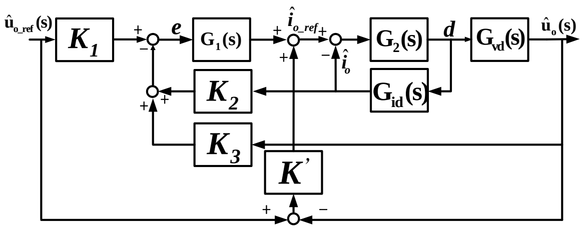

Fig. 6: The closed loop structure of circuit with game theoretic control

The equations of control are

$$\hat{i}_{o\_ref}=K\frac{1}{s}\times(k_1\hat{v}_{o\_ref}-k_2\hat{v}_o-k_3\hat{i}_o)+K'(\hat{v}_{o\_ref}-\hat{v}_o) \tag{15}$$

$$\hat{d}=(\hat{i}_{o\_ref}-\hat{i}_o)(K_P+K_I\frac{1}{s}). \tag{16}$$

Combine the (15) with (16)

$$\begin{aligned} \hat{d}&=K\frac{1}{s}\times(K_1\hat{v}_{o\_ref}-K_2\hat{v}_o-K_3\hat{i}_o)+K'(\hat{v}_{o\_ref}-\hat{v}_o)-\hat{i}_o]\times(K_P+K_I\frac{1}{s}) \\ &=[(KK_1\frac{1}{s}+K')\hat{v}_{o\_ref}-(KK_2\frac{1}{s}+K')\hat{v}_o-(KK_3\frac{1}{s}+1)\hat{i}_o]\times(K_P+K_I\frac{1}{s}) \\ &=[(KG_1(s)+K')\hat{v}_{o\_ref}-(KG_1(s)+K')\hat{v}_o-(KG_1(s)+1)\hat{i}_o]\times G_2(s) \end{aligned} \tag{17}$$

where $G_1(s)=K\frac{1}{s}$, $G_2(s)=K_P+K_I\frac{1}{s}$, $K=\frac{2BK_2}{A}$.

Assuming the input voltage $U_{in}=20V$, the parameters are given as fallows, $K=0.01$, $K'=0.5$, $K_P=0.5$, $K_I=1$, $K_1=1$, $K_2=0.8$, $K_3=0.2$. The open loop gain of current can be given by

$$T_{k\_i}(s)=G_2G_{id}=\frac{6\times10^{-2}s^2+10.12s^2+20}{6\times10^{-5}s^3+10^{-2}s^2+20s}. \tag{18}$$

Also, the bode plot can be obtained as Fig. 7.

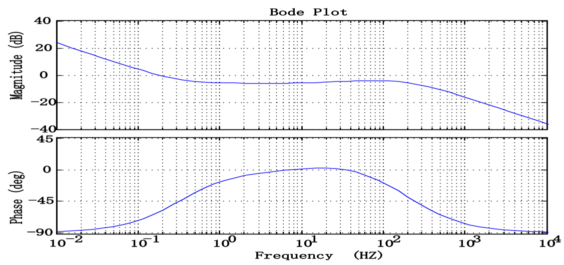


Fig. 7: The bode plot of current loop

As is shown in the Fig. 7, phase margin is about 110, the design of current loop is in a steady state. Therefore, the close loop structure with game theoretic control in Fig. 6 can be simplified as Fig. 8.

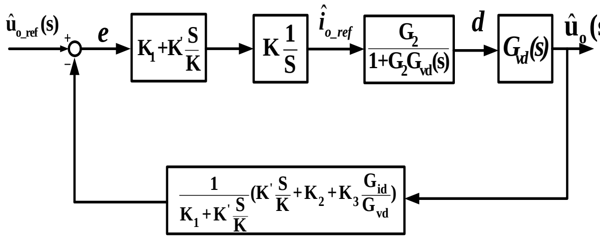

Fig. 8: The simplify close loop structure of circuit with game theoretic control

As is shown in the Fig. 8, the overall open-loop transfer function is obtained by

$$\begin{aligned} T_v(s)&=\frac{1}{K_1+K'\frac{S}{K}}(K'\frac{S}{K}+K_2+K_3\frac{G_{id}}{G_{vd}})\times(K_1+K'\frac{S}{K})\times K\frac{1}{S} \\ &\times\frac{G_2}{1+G_2G_{vd}(s)}\times G_{vd}(s)=\frac{G_2}{1+G_2G_{vd}}\times(K'G_{vd}+K_2K\frac{1}{S}G_{vd}+KK_3\frac{1}{S}G_{id}) \\ &=\frac{3\times10^{-4}s^4+5.06\times10^{-2}s^3+100s^2+208s+0.2}{18\times10^{-11}s^6+24\times10^{-8}s^5+18.536\times10^{-5}s^4+8.5\times10^{-2}s^3+30.12s^2+20s}. \end{aligned} \tag{19}$$

Through the overall open-loop transfer function, the bode plot can be obtained as Fig. 9.

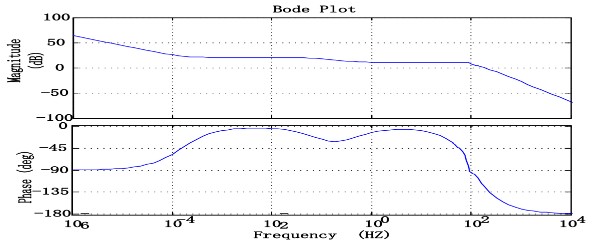


Fig. 9: The bode plot of overall open loop transfer function

From Fig. 9, it can be seen that phase margin is about 80, the system is in a steady state.

Through the small-signal model of the system control with game theoretic strategy, the design of the controller is stable.

## IV. Numerical Simulation

In this part, by using the concept of virtual rival strategy, a system composed of two DGs with a resistive load and a storage device is used for simulate the micro-grid system. In this paper, we adopt the buck converter as the main circuit of controller. One DG is consisted of photovoltaic generator and its controller, and the other DG is consisted of wind turbine and its controller. The simulation structure of simplified system of micro-grid is shown as Fig.10, and the parameters of the model are as follows: the output inductor is 10e-2 H, the resistance of load is 20 Ω , the switching frequency adopt 2 kHz.

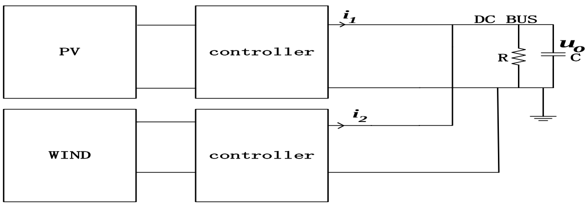


Fig. 10: Simulation structure of simplified system of micro-grid

### *A. Simulation Experiment*

According to the design parameters above and the reference value of the DC line bus voltage is 20V. The results obtained from simulations in MATLAB are represented in Fig. 11, Fig. 12 and Fig. 13which show the state of micro-grid with different control methods. The current of each DG is shown in the upper picture, and voltage of DC line bus is shown in the lower picture. At the upper picture, the two curves indicate the current of photovoltaic generator and wind generator circuit respectively.

When two DGs are adopted open loop control, the results of current share mainly depend on the symmetry of module parameters. As is shown in the Fig. 11, it is clear that the voltage is maintained in 20V and the performance of the current sharing is bad. Although using the open loop control can meet the voltage regulation, the performance of the current sharing is not ideal.

When the micro-grid system adopt the droop control strategy, as is shown in Fig. 12, it is clear that the output current of each circuit is almost the same, and the error of current is less than 5%. However, the voltage of DC bus is only maintained about 17V which causes a drop in voltage regulation. Although using the traditional droop strategy can meet the request of output current sharing, the regulation of DC line bus voltage is not ideal.

Compared with the traditional droop control strategy and directly parallel strategy, the game controller has a good performance on voltage regulation and current sharing. As is shown in the Fig. 13, it is clear that the output current of each circuit is almost the same and the voltage is maintained in 20V. In addition, the regulation time of game system is as long as the traditional controllers which adopt the droop strategy. In conclusion, the game theoretic method is possible to control each system rapidly and independently.

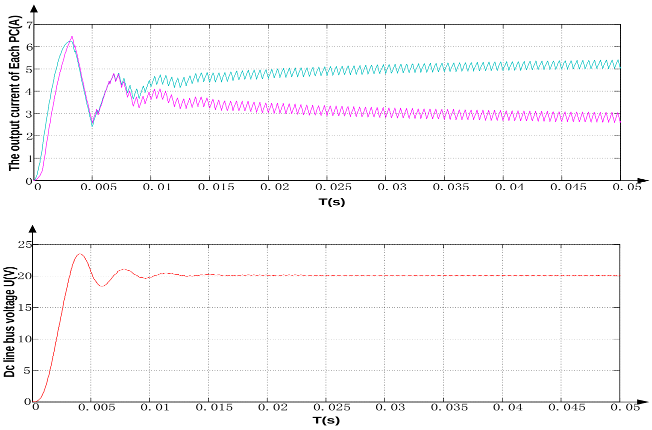


Fig.11: State of micro-grid system with the open loop control strategy

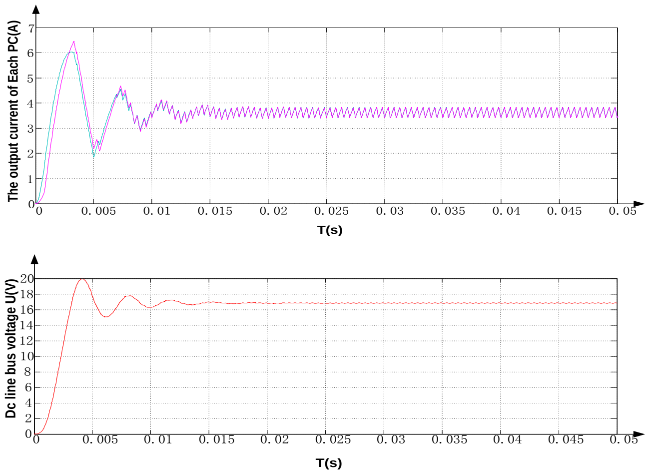


Fig.12: State of micro-grid system by droop control method

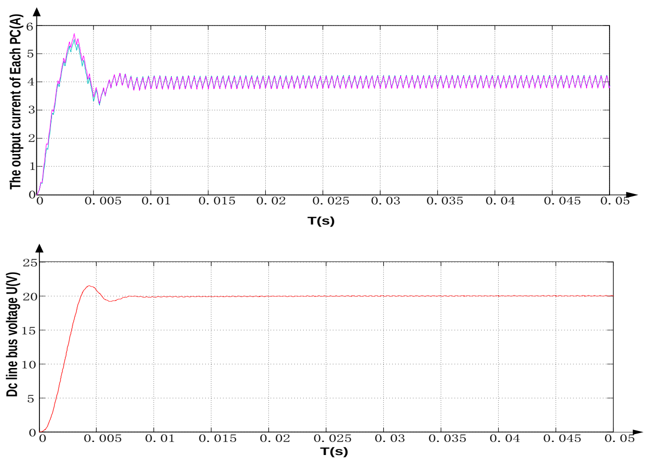


Fig.13: State of micro-grid system by game theoretic strategy method

*B. Dynamic Response*

In order to study the controller performance to large transients, a variable load is implemented (a load of 10 Ω is turned on in the micro-grid output). In this part, the reference value of the DC line bus voltage is 24V. The Fig. 14, Fig. 15 and Fig. 16 show the load change in the micro-grid when time arrive at 0.05s.

Fig. 14, Fig. 15 and Fig. 16 present the experiments for a load step from 100% to 50% of the output power. During this transient, an overshoot of less than 5% is observed in the output voltage with a good regulation as required. At the same time, the current of each DG changes synchronously, which achieves current sharing state finally.

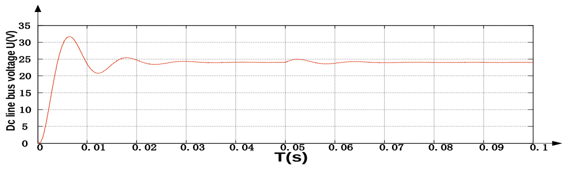


Fig.14: Voltage of dc line bus when load change in 0.05s

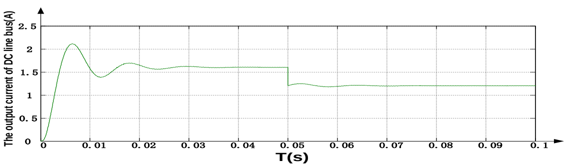


Fig.15: Current of dc line bus when load change in 0.05s

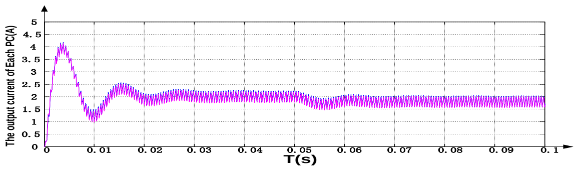


Fig.16: Current of each PC when load change in 0.05s

*C. Experimental Result*

An experimental result is produced by laboratory prototype, as shown in Fig. 17. This prototype consists of two PV simulators, and their buck converters connected to a stand-alone DC system which has an output storage ultra-capacitor and an adjustable resistive load. The simplified system is shown in Fig. 10.

The Fig. 17 and Fig. 18 show the measured waveform for a step load increase. 100% load is increased at t = 370s, causing a transient step up of the output current waveforms of two DGs. The experimental results of the microgrid voltage are depicted in Fig. 17, which shows the change of the DC bus voltage due to the variations of the load. Fig. 18 shows the output current of each DG converter. During the dynamic process, the current sharing is achieved by the two DGs. The measured current waveforms in Fig. 17 show the rapid response of the current-regulated. At the same time, the voltage of DC bus keeps the reference value stable in 24V.

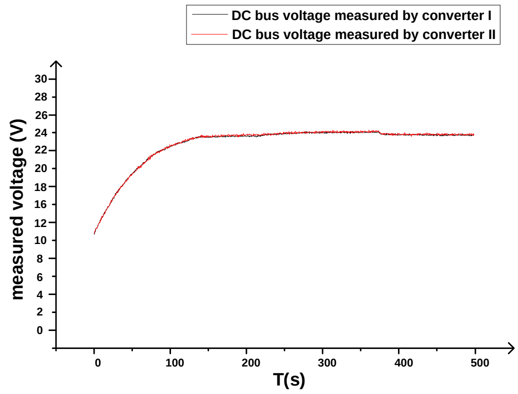


Fig. 17: Measured voltage of each converter when load steps increase at 370s

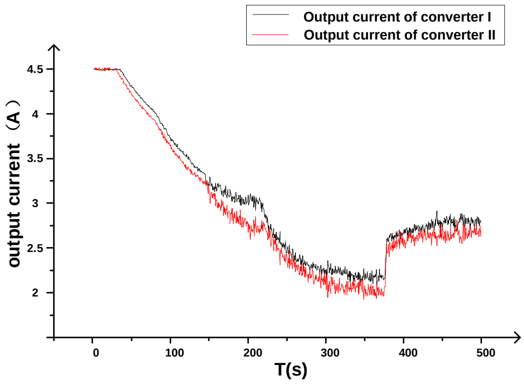


Fig. 18: Measured output current of each converter when load steps increase at 370s

## V. CONCLUSION

In this paper, the game controller is applied to DGs and it can be seen that using this method achieves desired performance in voltage regulation and current sharing among the micro-grid system. In order to simplify the analysis, we adopt the concept of virtual rival strategy that simplifies the n-player-game problem to two-player-game problems.

A small-signal stability analysis of the proposed control approach was shown. In addition, the proposed approach has been tested extensively in simulation. The controller allows DGs to share power generation without communications, and the performance of dynamic response is obtained by experimental results. It is concluded that the new control strategy shows good results in transient issues, power sharing, and stability.

## REFERENCES

[1] H. Kakigano, Y. Miura, and T. Ise, “Low-voltage bipolar-type DC microgrid for super high quality distribution”, *IEEE Trans. Power Electron.*, vol. 25, no. 12, 2010.

[2] P.N. Enjeti, P.D. Ziogas and J.F. Lindsay, “Programmed PWM Technique to Eliminate Harmonics: A Critical Evaluation”, *IEEE Trans. Ind. Appl.*, vol. 26, no. 2, pp. 302-315, March/April 1990.

[3] Y. Xiao, B. Wu, S. Rizzo and R. Sotuden, “A Novel Power Factor Control Scheme for High Power GTO Current Source Converter”, Conference Record IEEE-IAS, pp. 865-869, 1996.

[4] A. L. Dimeas and N. D. Hatziargyriou, “Operation of a multiagent system for microgrid control”, *IEEE Transs on Power Systems*, vol. 20, no. 3, AUG. 2005.

[5] K. Border, Fixed Point Theorems with Applications to Economics and Game Theory. Cambridge, U.K.: Cambridge University Press, 1985.
[6] S. Barsali, M. Ceraolo, P. Pelacchi, and D. Poli, "Control techniques of dispersed generators to improve the continuity of electricity supply", *Power Engineering Society Winter Meeting*, vol.2, 2002, pp.789 – 794.
[7] K. D. Brabandere, B. Bolsens, J. Van den Keybus, A. Woyte, J. Driesen, R. Belmans, K.U. Leuven, "A voltage and frequency droop control method for parallel inverters", Power Electronics Specialists Conference, vol.4, pp. 2501 - 2507.
[8] T. L. Vandoorrn, B. Meersman, L. Degroote, B. Renders, and L. Vandevelde, "A control strategy for islanded microgrids with DC-link voltage control", *IEEE Trans. on Power Delivery*, vol. 26, no. 2, 2011.
[9] D. Fudenberg and J. Tirole, Game theory. MIT Press, 1991.
[10] Mingming Chen, Advanced Mathematics, Beijing: Chemical Industry Press, 2010-2011.
[11] D. Smart, Fixed Point Theorems. London, U.K.: Cambridge University Press, 1974.
[12] J. Kim, Guerrero, J.M. Rodriguez, P. Teodorescu, R.K. Nam, "Mode adaptive droop control with virtual output impedances for an inverter-based flexible AC microgrid", *IEEE Trans. Power Electron*., pp. 689 – 701, 2011.
[13] A. Haddadi, A. Shojaei and B. Boulet, "Enabling high droop gain for improvement of reactive power sharing accuracy in an electronically-interfaced autonomous microgrid", Energy Conversion Congress and Exposition (ECCE), pp. 673 – 679, 2011.
[14] L.B. Perera, G. Ledwich, A. Ghosh, "Distribution feeder voltage support and power factor control by distributed multiple inverters", Electrical Power and Energy Conference (EPEC), pp. 116 – 121, 2011.
[15] Avelar, H.J, Parreira, W.A, Vieira, J.B, de Freitas, L.C.G, Coelho, E.A.A, "A State Equation Model of a Single-Phase Grid-Connected Inverter Using a Droop Control Scheme With Extra Phase Shift Control Action", *IEEE Trans. on Ind. Electronics*, pp. 1527 – 1537, 2012.
[16] A. Y. Wang, J. Huang, C. Wu and H. Mohsenian-Rad, "Demand side management for wind power integration in microgrid using dynamic potential game theory", *GLOBECOM Workshops* (GC Wkshps), pp. 1199 – 1204, 2011.
[17] B. Oraw, R. Ayyanar, "Small signal modeling and control design for new extended duty ratio, interleaved multiphase synchronous buck converter", Telecommunications Energy Conference, 2006, pp. 1– 8, 2006.

## BIOGRAPHIES

**Mingxuan Wu** obtained his Bachelor degree and master degree from Beijing Institute of Technology. He is currently an engineer in the Department of China Southern Power Grid. His research interests are in the areas of DC microgrid system and new energy power generation.

**Lei Dong** was born in 1967. He is currently a professor in Beijing Institute of Technology.His research directions are power electronics and power drive,and new energy power generation.

**Furong Xiao** obtained his bachelor degree from the Beijing Institute of Technology in 2011,and still study for the PHD degree in this school. His research direction is power energy conversion and micro grid.

**Xiaozhong Liao** was born in Guangdong, China, in 1962. She received her BSc and PhD degrees respectively from the Tianjin University in electrical and automation engineering and the Beijing Institute of Technology in control engineeering. She is currently a professer in Beijing Institute of Technology. Her research intersts are the electric drive system and energy converter control.